THE MATHEMATICS TEACHER

The Official Journal of
The National Council of Teachers of Mathematics
Incorporated 1928

FEBRUARY 1936

« Contents »

Volume XXIX **Number 2**

Published by the

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
MENASHA, WISCONSIN NEW YORK

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER is published monthly except June, July, August and September. The subscription price is $2.00 per year. Single copies sell at 40 cents each.

Miss Martha Hildebrandt

New President of the National Council of Teachers of Mathematics

THE MATHEMATICS TEACHER

Volume XXIX Number 2

Edited by William David Reeve

The Organization of a Course in General Mathematics for the Seventh Grade

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DURING THE session 1932–33 four of the seventh grade mathematics teachers in Albemarle County, Virginia, worked with the writer in organizing a general mathematics course for the seventh grade. The study was made possible through the coöperative arrangement existing between these schools and the department of education of the University of Virginia. Three projects were undertaken: First, a systematic and comprehensive analysis was made of the objectives of seventh grade mathematics as controls of content. Second, a catalog of seventh grade pupils' interests was compiled to serve as a point of departure in introducing the units of work to the pupils. Third, the difficulties of seventh grade pupils in common and decimal fractions were ascertained as a basis for remedial work. In its final form, the course contained ten units of work.

DETERMINATION OF OBJECTIVES

In determining the list of specific aims the first step was to record those appearing in the St. Louis Course of Study for the seventh grade. Next, the seventh grade aims found in four other courses of study[1] were analyzed. Each aim of the St. Louis list which was found duplicated or implied in the aims of any of these four additional sources was checked. Also, any aim found in them and not appearing in the St. Louis list was added and checked for frequency of appearance in subsequent sources. This temporary list was finally checked with Schorling's *Tentative List of Objectives for Junior High School Mathematics.* No aims were added to the list from this last source, since it was not possible to decide which of these aims are intended for the seventh grade and which for grades eight and nine.

The aims were then classified under eight headings with a basic and secondary list for each. Any aim occurring in four of the five courses of study or two of them and the Schorling list was placed in the basic group. Other aims were placed in the secondary list.

The opinions of the teachers and writer were next utilized in validating the aims. Those in the basic list were studied first. If four of the five members of the committee thought an aim was basic to seventh grade mathematics, it was retained. In the same manner, the thought of the group was determined as to whether or not any aim was more appropriate for the eighth grade. Likewise the secondary

[1] Pittsburgh, Pa., West Virginia, Fresno, Calif., Muncie, Ind.

list was studied to decide which should be classified as basic, which left for the eighth grade, and which eliminated.

Eight general mathematics texts for the seventh grade were next analyzed. In this analysis, each aim of the original list was checked against the content of each text. Moreover, any item of content which was not included in the aims was added as an aim which would utilize the specific item of content.

Determination of Pupil Interest

To investigate the interests of seventh-grade pupils a suggested list of items was submitted to the teachers cooperating in the study. Pupils were asked to indicate "Much interest," "A fair amount of interest," "No interest," in the items of the first four parts of the list. The fifth part included a number of vocational activities to be checked for first, second, and third choices. Additional items were added by the pupils to denote special interests. Returns were secured from 104 pupils. On the basis of frequency of "Much interest" checks the items were ranked for each of the four parts and the highest twenty per cent selected; these appear in Table I.

Table I

The Highest Twenty Per Cent of Items in which 104 Seventh Grade Pupils Evidenced "Much Interest"

	Much interest	*Fair amount of interest*	*No interest*[2]
PART I—PEOPLE			
1 Cowboys	62	17	24
2 Aviators	58	32	13
3 Explorers, Discoverers	53	31	12
4 President of the United States	51	25	27
5 Detectives	48	32	22
6 Sailors	47	35	19
7 Inventors, Scientists	46	37	10
8 Boy Scouts	42	35	30
9 Nurses	40	33	30
10. Soldiers	34	41	27
PART II—NATURE			
1. Birds	86	11	6
2. Stars	79	17	8
3. Domesticated Animals	75	19	9
4. Sun	73	26	5
5. Moon	71	28	4
6. Eclipses	63	31	9
7. Vegetables	63	29	10
8. Mountains	63	28	12
9. Lakes, Rivers, Oceans	60	34	10
10. Trees	56	38	9
PART III—MATERIAL INTERESTS			
1. Radio	84	16	4
2. Automobiles	84	16	3
3. Money	80	15	8
4. Clothing	77	21	6
5. Fruit	75	27	1
6. Airplanes	73	21	9
7. Books	72	23	8
8. Balls	72	22	10
PART IV—ADAPTIVE INTERESTS			
1. Movies	75	18	11
2. Baseball	64	23	17
3. Historical Places	62	36	5
4. Programs	60	34	10
5. Inventions	61	34	9
6. Water	60	34	10
7. Electricity	58	27	18
8. Tennis	56	18	11
9. Girl Scouts	55	20	28
10. Football	48	32	24
11. Printing	46	42	15
12. Planting Seed	44	43	16

From Part V containing twenty-three vocational activities the five receiving the greatest number of "first-choice" checks were: (1) care of animals, (2) recording and systematizing records, (3) operating machines, (4) scientific work, and (5) growing plants. These, with the items in Table I, were considered significant interests to be drawn upon in introducing the units of work determined by the analysis of aims. Such an introduction written for the unit on "percentage" is given to illustrate this use of the tabulated interests.

In your previous work in mathematics, you have learned much about the topic of this unit; percentage. Much of this work, though, included exercises in how to change from hundredths to per cents and vice versa, how to find the per cent of a number, etc. In this unit of work you are going to extend your knowledge of percentage and see how, without it, you could not do many of the things boys and girls do of their

[2] The horizontal totals in this table are not all 104 for two reasons; first, the irregular attendance of pupils on the several days the check list was used and, second, the appearance of certain "special interests" added by the pupils and not checked by all.

own accord as well as many of the things that men and women do in after-school life.

Suppose you should select as a hobby to occupy your leisure time a study of birds. James Lane Allen made such a study of a particular kind of bird and put the results of his observation in a most entertaining little book: *The Kentucky Cardinal.* Now suppose, after you had observed a particular kind of bird or a number of mixed varieties, you wanted to tell someone about what you found. How would you tell a friend what portion of the eggs laid actually hatched? How would you say the food of the birds observed was divided in amount among several things, such as worms, weed seed, insects, etc.? How would you compare the rate of increase or decrease in numbers of one variety of bird with another? Would you not find percentage the most effective means of making these explanations?

Suppose, again, that you undertook for the summer vacation a project such as raising chickens, pigs, or tending a vegetable garden. How would you explain the relationship of your gain or loss to your expenses? If you graded the eggs from your hens, how would you explain the relationship of culls to primes; how would you compare the number of chicks hatched to the number of eggs set? Would not percentage serve your end again?

You are soon to find out in this unit that the costs of many articles such as radios, automobiles, clothing, books, and vegetables all change. The rate of increase or decrease is almost invariably expressed in terms of percentage. Moreover, in the manufacture of these articles, the relationship of cost of labor, cost of raw products, cost of delivery, cost of overhead, and other items, to each other and to the entire cost is expressed in terms of percentage.

We all know that accuracy is an extremely important characteristic in all scientific work. Suppose a doctor prescribes a ten per cent solution of some medicine for a patient. Must not the nurse or druggist know exactly how to determine this percentage and be accurate in his determination? A stronger solution might prove fatal to the patient. All scientific work, though, based on measurement is inaccurate to some degree. Scientists spend much of their time and energy in reducing the "error of measurement" in their work. They determine the *percentage* of error in every piece of work and are happy when this is very low.

Finally, percentage finds use in baseball and the movies. Do you know what per cent of the price of a movie ticket is tax? Would you be interested in knowing what per cent of the population at Hollywood is composed of actresses and actors? What do you think is meant by the statement, "The batting averages of the four leading hitters in the American League are: 413, 396, 385, 376"? Here we have percentage again. "413" means the leading batter in the American League is getting hits 41.3% of the chances he has to hit.

You readily see from this discussion that percentage has almost an endless number of uses. When you have finished this unit you should have confidence in undertaking anything that involves percentage.

Analysis and Correction of Fraction Difficulties

At the very beginning of the session, diagnostic tests in common and decimal fractions were given the pupils included in the study. These tests contained five parts. Each of the first four parts was on one of the fundamental operations with fractions and the fifth contained verbal problems involving fractions. An idea of the comprehensiveness of these tests may be had from the fact that the common fraction test alone examined 56 skills in addition, 36 in subtraction, 46 in multiplication, and 8 in division.

The results of this diagnosis were used as guides to exercises placed in the units and designed definitely to correct the weaknesses revealed. The results showed that pupils may be efficient in performing the operations with fractions and yet fail to deal with them correctly in problem situations. Therefore, the remedial exercises selected for the units were not isolated from the content of the unit proper, but on the contrary every effort was made to effect the correction through the new problem material. That is, in every situation in which common or decimal fractions could be used they were introduced. Parts of typical units are quoted to illustrate the use of the corrective exercises. The first is taken from the unit on "Measuring Rectilinear Figures," the second from "Making Geometric Constructions."

A. Exercise for notebook:

1. Find the areas of triangles with following altitudes and bases:

Altitude	Base
3.4 in.	12.7 in.
17 in.	37.3 in.
.25 in.	8.8 in.

2. Find the bases of triangles with following areas and altitudes:

Area	Altitude	Area	Altitude
$12\frac{1}{3}$ sq. in.	$3\frac{5}{8}$ in.	13.75 sq. ft.	5 ft.
$27\frac{1}{2}$ sq. in.	$5\frac{1}{2}$ in.	23.05 sq. ft.	2.31 ft.
24 sq. in.	$7\frac{1}{3}$ in.	51.2 sq. ft.	3.2 ft.

B. Exercises for notebook:

1. Change to degrees and minutes: $30\frac{1}{2}°$, $20\frac{3}{4}°$, $19\frac{5}{8}°$, $125\frac{3}{8}°$.
2. Change to degrees and fractions of a degree: 160′, 936′, 1850′, $1552\frac{1}{2}'$.
3. How many minutes in $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $1\frac{1}{3}$ of each of the following angles? 240′, 27′, 54′, $2\frac{1}{2}°$, 15′, 45″, 3°, 40″, 15′.
4. What fractional part of a degree is each of the following angles? 14′, 35′, 28′, 120″, 47′, $22\frac{1}{2}'$, $37\frac{1}{2}'$.
5. Make following divisions to nearest tenth:
   $3\overline{)29°14'}$, $5\overline{)71°12'}$, $8\overline{)67°9'}$, $6\overline{)57°15'}$
6. Change angles in (1) and (2) to degrees and nearest hundredth of a degree.
7. Find average of eleven angles listed in (4) and (5).
8. Take "Improvement Test No. 10," p. 203, *Strayer-Upton*.
9. Class exercise: draw, with your protractor, angles of $45\frac{1}{2}°$ and $115\frac{1}{2}°$, then bisect them with ruler and compasses; check your construction by calculating the number of degrees in the half of each angle and then measuring the halves.

Summary

The advantages claimed for a course in high school mathematics planned as outlined in this article are: first, it fixes in advance, by objective methods, the content to be included and rejects pupil interest as a control of content; second, it utilizes pupil interest in motivating the work of pupils on content that has educative value; and third, it corrects the characteristic deficiencies in common and decimal fractions, not through just so much more drill, but through the functional use of fractions in the numerous new and interesting situations found in seventh grade general mathematics.

The Place of Arithmetic in Modern Education

One of the ancients said: "*Remember, O Stranger, Arithmetic is the first of the sciences and the mother of safety.*" History has never been able to contradict that statement, and yet today, in America, in Catholic education and non-Catholic education, arithmetic is the most neglected of all sciences. Today a child's study of arithmetic stops at the age of 12, and many of our ills today can be attributed to the fact that we are trying to live men's lives and a great nation's life based upon a child's understanding of arithmetic. Let us in Catholic education carry the study of arithmetic right straight through the high school and through the college, through business arithmetic, bookkeeping, and cost accounting, algebra and geometry, in so far as they supplement the understanding of the highest arithmetic, cost accounting, values and so forth, so that when a boy or girl comes out of our high schools or our colleges he or she will be able to not only budget his income and his outgo, but to break down that budget and bring home to himself a realization, for example, of what cost prohibition, what cost foreign trade, what cost a credit system controlled abroad, what cost corruption in government—national, state and city—what cost tariffs, high or low, what cost crime and its punishment, what cost the administration of our charities, as well as what cost his home—how much of that cost is usury—how much of that cost is dishonesty of construction or dishonest materials—what cost neglect of our own health or the health of our children, and so forth and so on.—From a speech by Francis P. Garvan before the Friends of the Catholic University of America at a Dinner at the Knights of Columbus Club Hotel on February 1, 1933.

Quantitative Thinking on the Secondary School Level

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Introduction. A curious paradox is the contrast between the striking success of mathematical techniques in forming a foundation for, and a method of constant improvement of, our whole industrial civilization; and the complete failure of conventional mathematics to develop any facility in quantitative thinking among the great mass of American people.

Mathematics teachers should not be blamed too much if our people are quantitative illiterates. They have taught arithmetic, algebra, geometry, and trigonometry so that their students might pass tests such as the New York Regent's or those of the College Entrance Examination, or some other similar test. They have stressed pre-professional mathematics and the type of quantitative thinking involved therein which is of great value to the scientist and engineer but of little value to the layman.

Mathematics teachers no doubt will say that their purpose has been to train a select few, not the masses, in these special types of thinking. No doubt many, if not most, of such teachers will take the position that the great mass of people cannot be educated so that they will think quantitatively in everyday affairs. In terms of the present content of mathematics these teachers are right, but what if we set up our objectives first and make our content fit these objectives? We need to make a sharp distinction between the logically organized classical divisions of mathematics which come from a dim and distant past and which are absolutely essential to certain specialists, and those concepts and principles of mathematics which might form the basis of quantitative thinking for everyone in his social and economic and political affairs. Before we take up the question of whether or not it is possible to educate every citizen for quantitative thinking which he needs for intelligent social behavior irrespective of his occupation, let us look at conventional mathematics a little further.

That mathematics can continue to justify itself on the basis of a classical tradition does not seem probable. Neither the traditional nor the classical views seem to be adequate in terms of present American living. In fact they appear to be completely sterile and irrelevant if not entirely discredited. The reasons for this state of affairs is not hard to find.

The conclusions from a long list of experimental studies on the extent of transfer of training are not very comforting to the enthusiasts of formal discipline. Nothing convincing has yet been brought forth to show that the worthy qualities attributed to mathematics training transfer to situations not involving similar mathematical situations. There is nothing to show that those who have mastered mathematics approach the problems of life with any more social intelligence and judgment than those of the same level of general intelligence who have not been through such courses. Closely related to this point is Downing's investigation of the scientific thinking of science and non-science pupils in the high school. He found no significant advantage for the pupils who have studied science; they did no better in their thinking than those who had taken no science courses whatever.

Nor can we continue to appeal to the mystical influence of mathematics to develop such mental powers as "reasoning," "memory," "imagination," or "judgment" although this view still seems to be widely held among both educators and laymen. Faculty psychology has long been discredited, and anything which is based

upon it rests upon a very flimsy foundation indeed. The investigations of Lashley and others in the field of brain structure and functioning seem to point to a generalized functioning of the brain with regard to mental processes. Certainly there appears to be no special brain area devoted to reasoning which is developed alike by geometry, ancient history, or Latin. On the other hand skill in thinking cannot be divorced from content, as many studies on discovery and invention clearly reveal.

We cannot make a very strong case for mathematics on the basis of the social frequency of certain mathematical concepts. Just because quantitative terms and symbols occur in magazines and newspapers is not a reasonable justification for teaching mathematics in the high school. It appears from Bobbitt's investigations that these terms and quantities are not very numerous; all of them could probably be explained in a handy reference manual to sell for half a dollar, and require at the most about a month or so of study to master. Mathematics must be put on a firmer and stronger foundation than this if it is to have a place in the secondary school curriculum.

Nor can mathematics in a high school designed for purposes of general education be justified on the basis of its vocational uses, however important and numerous the latter may be. As Downing has pointed out with reference to science, we have been emphasizing technical and producer science when we should have been emphasizing consumer science. The former, by its very nature, must be limited to a relatively few specialists; the latter includes everyone regardless of his occupation. Strictly speaking then only consumer mathematics is eligible to become a part of the content of general education.

We cannot overlook the widespread existence of social, economic, and political illiteracy among our people. Numerous observations and investigations could be cited to illustrate this point. As long as social ignorance is rampant, it ill behooves us to maintain in the curriculum subject matter which does not directly contribute to the elimination of this deficiency, which is largely a carry-over from a dim and distant past, and which has been carried forward by an aristocratic tradition no longer tenable.

Most of the reasons given for the study of mathematics appear to be little more than rationalizations, or justifications of why we should continue in the path over which we have always traveled. Strange as it may seem, the reasons for studying mathematics which are put forward by some subject matter specialists in this field, could with equal cogency be used to defend the teaching of such subjects as phrenology, theology, or Arabic. Nor is this attitude, we hasten to add, peculiar to teachers of mathematics alone.

The basis of reorganization. In order to create an adequate mathematics for the purpose of general education at the secondary level, we believe the following conditions at least must be met:

1. Mathematics must stress its unique purpose: quantitative thinking for all.
2. Mathematics must deal with those quantitative aspects of greatest value in developing the socially intelligent citizen regardless of occupation.
3. Content must be selected therefore, not on the basis of some logical organization such as Euclid's, but upon the extent to which it contributes directly to socially intelligent living.
4. Mathematics for general education must be divorced sharply and completely from mathematics for pre-professional education. The latter should still be offered for the training of those specialists who require it.

A word about each of the foregoing statements. Mathematics is the only field which deals with quantitative thinking; that is the unique principle underlying its organization. In a quantitative age such as ours, quantitative thinking is just as important as ordinary linguistic thinking; in fact one may argue that it is more important. Without pressing the point, we believe that quantitative thinking

worthy of the name is no more complex than ordinary thinking which would be necessary to understand Shakespeare or Emerson, or follow a speech by Herbert Hoover or Franklin Roosevelt. We therefore interpret quantitative thinking to include those techniques and principles which are just as necessary for ordinary living as language itself. Therefore, experience with this type of thinking will be required of all students.

A corollary of our second statement is the following: Mathematics must deal with those principles and techniques which will make the individual more accurate and more rational in his thinking and behavior. We make the assumption here, and we have some support for it from observation and investigation, that one reason why citizens are not as socially intelligent as they might be is that they are quantitative illiterates.

If one accepts the first two statements then the last two statements follow as applications of the first two principles. It will be noted that mathematics would consist of two sets of courses: those involving quantitative thinking for all, and those similar to our present courses designed as foundation courses for technicians.

Quantitative thinking for all. In order to clarify the meaning of "quantitative thinking" we shall first contrast mathematics for general education with present mathematics. In the former there will be more discussion and less computation, more insight into principles and problems and less drill, more content from the social sciences and less from the physical sciences, more stress on real individual and social problems and elimination of irrelevant and fictitious problems, more stress on social utility and elimination of content for its own sake, more stress on practising quantitative thinking and less on just going through motions in order to pass a test.

Our social science teachers avoid the quantitative aspects of social problems while our mathematics teachers avoid the social implications of quantitative principles. There is a real need to bridge this gap, a process which we think will add vitality to mathematics and accuracy to the social sciences. About the only course we now have which attempts to bridge this gap is social statistics which is offered on the college level, but never on the high school level.

No doubt many will say that such a course will be too difficult; that students will be unable to understand quantitative principles let alone apply them. Here again we say that much depends upon the principles, the way in which they are presented in books, and the type of teaching method which is employed. Another fact which cannot be overlooked in this connection is the strong trend to push courses ever downward from the college into the high school, and from the high school into the junior high school. Many of the courses now offered in the high school were once offered only on the college level.

Illustrations of content. While the writer cannot present at this time the details of the various courses in quantitative thinking which might be offered on the secondary school level, he does have at hand several illustrations of the types of problems which might be treated in such courses. These examples are offered in order to give a more definite meaning to what we have been discussing; they do not cover the range of possibilities nor do we guarantee that they represent the best illustration in any particular case.

Accuracy. The Chicago *Tribune* for December 18, 1932 reports the results of a study carried out by a group of anti-prohibitionists who found that prohibition had cost the American people in 12 years the total of $34,565,109,246. This same paper reported that 3,883,891 people had been killed in U.S.S.R. in 14 years although most of the items entering into this total were crude estimates if not outright guesses. The New York State Department of Education announced in 1935 that the two students receiving the first

and second places on the State list of scholastic ability obtained averages of 99.333 and 99.142 respectively.

Averages. Many people make a fetish of "underweight" and "overweight" feeling that if they weigh a few pounds less or a few pounds more than the usual norms dire consequences are bound to follow. This belief needs to be de-bunked by showing how the usual norms are computed, what the probable error is, what fluctuations from the average weight occur among healthy people, and the difference in norms for healthy individuals who grow at different rates and mature at different ages.

The Associated Press recently carried a news report from Cleveland to the effect that athletics might make a boy smaller than he otherwise would be. The data were from a study of two groups of 30 boys, one called "non-athletes" and the other "athletes," the former group gaining 18 pounds in weight and 3 inches in height whereas the latter group gained 17 pounds in weight and 1 inch in height. The two groups were not equated for height, for weight, for anatomical age, for growth rates nor for any other significant factors and therefore the average gains proved just exactly nothing about the effect of athletics upon growth.

Per cent. One of the Chicago papers in 1934 carried the following statement in connection with an editorial defending the current tariff law: "In these circumstances there is no opportunity for vastly expanding American exports, which at the peak constituted *only 6 per cent* of the gross national income."

Another news item from the national capitol on April 26, 1934 ran as follows:

> Washington, D. C.: Workers and railroads arranged today to give back to the workers the 10% that had been cut from their wages thus averting the danger of a general strike. . . . The railroads will increase wages 2½% July 1; 2½% January 1, 1935 and 5% April 1, 1935 *thus putting them back where they were in 1931.*

The president of a flower growers' association announced in a newspaper interview: "Flowers are 100 per cent cheaper than four months ago."

Rates. In 1910 the death rate per 100,000 population in New York City and Richmond, Virginia, were respectively 187 and 226 leading one to suppose that the latter city was not so safe to live in as the metropolis. Analysis of these death rates according to white and Negro population showed a different situation: for white population, 162 for Richmond and 179 for New York City, while for the Negro 332 for Richmond and 560 for New York City. Due to the fact that more than half the population of Richmond while only 2 per cent of New York City's population were Negroes, this gave a general death rate favorable to the metropolis when the opposite was the real state of affairs.

A usual practice among those who desire to show that workers are pretty well off is to point to the amount of wages received per hour, or the increase in the wages received per hour over a period of several decades. They overlook the obvious fact that the total number of hours worked per week or year must be considered, since it is quite common that those who receive high rates of pay often are unemployed a substantial part of the year. Because a man is receiving a high rate of wages does not mean that he is receiving a decent yearly income.

Sampling. The columnists who advise the lovelorn have a habit of asserting that "men" feel and behave in certain ways, and that "women" feel and behave in certain other ways. It is probably a fair estimate that these columnists do not know with any degree of understanding more than 50 of each sex yet they dare to speak for over 30 millions in each group in the United States alone.

A few years ago the newspapers were telling us about a suicide wave among college students. Cases everywhere both here and abroad where the cause of death was not very clear were seized upon. In the midst of this excitement a statistician

for one of the life insurance companies presented facts to show that the rate of suicide among those under twenty years of age had been decreasing for sixteen years. Many "waves" appear to have a similar newspaper origin.

Mr. W. J. Reilly in a little book entitled "Straight Thinking" (Harpers) gives an excellent example of the dangers to social intelligence of a very common type of sampling:

> At a luncheon a friend recently told me that he had just returned from a trip throughout the Middle West and he proceeded to explain the "attitude of business leaders" in that section of the country toward "Roosevelt's Recovery Program." Toward the end of the conversation I found that my friend's trip throughout the Middle West consisted of three days at the Chicago Fair, one day in Cincinnati, and one day in Pittsburgh, and involved personal contacts with about a dozen men who were primarily interested in the tobacco business.

The straw ballot is a method of sampling the voters in order to predict the outcome of the election. Unless carefully done they cannot be trusted. Although *The Literary Digest* has had a high degree of success with its ballots, still it went somewhat astray in 1934 under-estimating the Democratic vote and giving Sinclair of California only 25 per cent of the vote whereas he actually received 38 per cent of the ballots cast.

Trends and prediction. It is not uncommon to hear people set up a population trend as a standard, and compare or contrast other trends with this one. At present it is common to point out the alarming growth of government debt, or taxes, or civil service workers, or the cost of education when compared with population growth. There is no particular reason why one should accept population growth, or any other trend as a standard of comparison; certainly not until adequate reasons are given for such a step. Many variables at the present time are at different points on their respective growth curves, and comparisons should be made with great caution, if at all. Taxes are increasing faster than population because we are demanding more and more from government particularly during recent years. There is no more logic in stating that taxes and cost of education should follow the population trend, than to assume, which most business men and bankers do not, that bank deposits and the production of automobiles and cigarettes should follow the population trend.

In his second annual message to Congress Lincoln predicted that our population would be 251,680,914 in 1930. Lincoln found that our population had increased up through 1860 at about a constant rate, but let him tell it in his own words:

> This shows an average decennial increase of 34.60% in population through the seventy years from our first to our last census yet taken. It is seen that the ratio of increase at no one of these seven periods is either 2% below or 2% above the average, thus showing how inflexible and consequently how reliable the law of increase in our case is. Assuming that it will continue it gives the following results. . . .
>
> (There followed a table of his predicted values.)

Professor Ogburn has pointed out how ludicrous some of our trend projections may become. Paraphrasing him we should say that if the death rate continues to fall during the next seven decades as it has during the last five, there should be nobody dying. If auto registration continues at the rate it increased during the 1920's, every family will have three cars in 1950. And if unemployment continues at the same rate during the next six or seven years that it has during the last two or three, no one will be employed.

Interpretation of data. We present here a few illustrations which we have found in recent months. In the New York *Times* for February 5, 1935 appeared the following Associated Press dispatch:

> According to the Census Bureau the population of the United States has jumped nearly 20,000,000 since 1930. The latest census estimate puts the population at 141,574,000 which compares with the 1930 census of 122,775,046. . . .

Clearly this is an error since our population does not increase at such a high rate. The former figure is the population of

United States and all its possessions whereas the latter figure includes only continental United States; hence the two figures are not comparable.

In the *National Republic* for April 1935 is an article dealing with primary elections from which was taken the following excerpt:

> Since the enactment of the primary election law, state taxes (in Minnesota) has (sic) increased several hundred per cent. Also the number of persons on official pay rolls has increased by several thousand.

No doubt here as to what is cause and what is effect!

George Seldes in his book "Freedom of the Press" states that the New York *Times* in the two years following the Russian Revolution reported that the Soviets were tottering to their fall exactly 91 times, that Petrograd fell 6 times, was on the verge of capture 3 times, and was burned down twice. It is only fair to say that the *Times* has mended its ways since then.

Colonel Frank Knox is reported to have made the following statement in a speech September 9, 1935:

> What about living costs? It now takes $1.80 in the dollar of today to buy in food and other necessities what you paid $1.00 for before March 1933.

If he said this, he was apparently confusing devaluation of the dollar with the purchasing power of money in domestic markets. The cost of living index of the United States Bureau of Labor Statistics shows the trend of purchasing power of the dollar: (1913 is 100)

June 1929	170	June 1932	136
June 1930	167	June 1933	128
June 1931	150	June 1934	136

Conclusion. Space does not permit us to give similar illustrations under other mathematical topics such as dispersion or variability, graphics, index numbers, correlation and association, probability, estimating, and checking, collecting data, and interpreting data. We hope, however, that we have given enough to show what we mean by quantitative thinking, why we believe that it should be divorced from pre-professional mathematics, and the type of principle from mathematics and the type of illustration from social and economic and political problems which might be employed as its content. Most of the quantitative thinking which comes to us now through the press and over the radio needs to be thoroughly de-bunked—and mathematics alone can do it.

A New Deal in Geometry

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One of the most striking developments in the teaching of geometry in recent years is the change that is taking place in the philosophy underlying the purposes of the course. When we read (1) "Our great aim in the tenth year is to teach the nature of deductive proof and to furnish pupils with a model for all their life thinking";[1] (2) "We believe that geometry may be made the basis of the study of the methods of reasoning which will be of use in any field where the necessary facts are at hand";[2] and (3) "He (the pupil) sees a sequence of theorems built up into a logical system and he sees how this system is constructed, the result being a basis of proved statements which he can use for establishing further proofs, precisely as a lawyer proceeds to construct his case or a speaker to construct an argument,"[3] we find that there is the *stated* objective, if not carried out in reality, which denotes a trend away from rote memory to the development of reasoning.

Heretofore teachers of geometry have been giving too much consideration to the application of the informational facts of geometry to life activities. My thesis is that there is not enough emphasis upon the processes of thought and reasoning used in the study of geometry, which can be applied to one's everyday experience.

Of course any new theory which involves additions to the present course naturally raises objections due to the time element.

With the reader's indulgence I shall discuss the "time element" so precious to educators in general, so essential to curriculum builders, and so necessary to the exponents of professionalized subject matter in mathematics.

We, in America, are "time" conscious. To go a step further, we are "four-year" conscious. Our high school is *"four years"*; our college undergraduate courses are *"four years."* Catalog after catalog of the Colleges and Universities throughout the country will designate the study of medicine, architecture, engineering, pharmacy, etc., whether undergraduate or post graduate, to be *"four years."* Truly a magic number it is, but as far as I can ascertain, without any scientific basis. This theory has been handed down to us historically and we are "stuck" with it. Nor is this the worst feature or significance of this heritage. We find that the subdivisions of the subject matter, in order to conform with the *"four year"* basis, must necessarily break up into *"one year"* groups. We therefore discover in mathematics, algebra "one year" and geometry "one year." This probably accounts for the reluctance of a large number of teachers and leaders in the field of mathematics to release their strangle hold on the "watertight compartment" division of the mathematics subject matter. In this discussion I am concerned with this "one year" idea of geometry. If we wish our pupils to acquire the ability to understand the need of a formal logical proof, to develop powers and habits of careful, accurate, and independent thinking,[4] and to make clear to the pupil the meaning of demonstration, the meaning of mathematical precision and the pleasure of discovering absolute truth,[5] what foundation have we from which to assume that he will have

[1] C. B. Upton, *Fifth Year Book of the National Council.* Bureau of Publications, Teachers College, Columbia University, 1930.

[2] Hassler and Smith, *Teaching of Secondary Mathematics.* Macmillan, 1930.

[3] Smith and Reeve, *The Teaching of Junior High School Mathematics.* Ginn and Co. Page 230.

[4] Hassler and Smith. *Ibid.* Page 297.

[5] Reeve, W. D., *Fifth Year Book of National Council.* Page 13.

reached the optimum ability at the *end* of one year? What scientific basis is there for the constant pounding of the inductive-deductive method of reasoning, day in and day out for the extent of *one year?* I have not been able to discover any. My search has been fruitless. If, on the other hand, optimum ability has been reached before the expiration of the usual period, and I believe it has, why do we waste the pupil's energy and valuable time beyond the margin of maximum return? Therefore before any suggestions can be constructively made whereby geometry may become more humanized, it is necessary that we agree to the argument that we spend too much time on the *same* kind of work in the present teaching of geometry. What is the answer?

1. *Diminish the number of propositions.* This does not necessarily mean that less concentration will be effected but in the words of Sinclair Wilson, "the emphasis should be not upon the ground covered, but upon the ground cultivated"; a succinct statement with a wealth of meaning. Many people will immediately reply that it will destroy the sequence. Let us see if history bears out this argument. A careful study of the textbooks and syllabi of the last thirty years shows a decrease in the number of propositions from 160 to 114, a reduction of about thirty per cent. This reduction has been brought about in many ways. Some of this discarded material is now treated informally or postulated for simplicity's sake. Others have been included among the exercises or stated as corollaries, while many, such as theorems on proportion, limits, projections, and triangles, extreme and mean ratio, constructions, properties of the pentagon and decagon, and the computation of the approximate value of pi, are no longer regarded as basic propositions. The sequence has not been affected by these changes. I propose a further reduction of the number of propositions, still retaining a firm substance so that the pupil will appreciate a logical sequence of a body of facts. What shall be discarded? I offer here a few suggestions:

1. Postulate the congruency theorems.
2. Postulate the similarity theorems.
3. Postulate any theorem which uses "superposition" (some Book II propositions).
4. Propositions that are not necessarily links in the sequence but are used only to lead up to computation of value of pi.

I can imagine the look of consternation on the faces of some of my readers. They may say: "But we cannot let these go, the pupils need them for college." True, in part, but what did our pupils do when we discarded the extreme and mean ratio problems? Those who went on to college had the mental ability to look it up and master them if necessary. What per cent of the pupils studying plane geometry will ever reach college? Should we arrange our materials specifically for the college entrant when only a small percentage enter college? The argument is without foundation and does not take into account that the trend of education today is away from college entrance preparation and towards a preparation for life. There is no reason why a body of say fifty propositions wisely chosen could not accomplish the same purpose as a sequence of 114 propositions. This would enable the consideration of a far greater number of original exercises, of simpler degree, of greater interest, and much better adapted to the purpose of having a larger number of pupils, who do not expect to be experts, appreciate the method of a demonstration.

2. *Develop a greater ability and insight into processes of thought, in addition to the inductive-deductive method.* The methods of proof as examples of processes of thought in geometry and applicable in every day life are the analytic, the synthetic, the indirect, the algebraic, and loci methods. In present day teaching the major emphasis is placed upon the first two methods. Very little time is given, if any, to the other three, and then only in

so far as they will satisfy examination purposes. As a matter of fact, some modern textbooks place the indirect method of proof at the end of the book in an appendix. Recent literature on the teaching of geometry has clearly demonstrated that the indirect method is the method most widely applied in everyday life activity. Professor Upton's masterful discussion in the Fifth Yearbook of the National Council of Teachers of Mathematics brings home, in no uncertain way, the importance of this process of thought and the manner in which it has been neglected in our teaching. No teacher of mathematics should fail to read this magnificent exposition of a vital method of thinking. To do anything but scratch the surface of this everyday method of thought would require an elementary introduction into the simple notions of logic. And why not? If a greater exposure to the indirect method will lead the pupil to a better and fuller appreciation of the proof as applied to life situations and activities, then it is essential that we teach those necessary and sufficient notions of logic prerequisite to a clear understanding of the method. Professor Upton rightly says that "we are absolutely dependent upon this method not only to establish a connected chain of propositions in geometry, but also in many life situations.[6] . . . If Mr. Jevons' statement is correct that nearly half of our logical conclusions rest upon its employment, then the time spent upon the indirect proof in the geometry class will be an investment of permanent value.[7] It is therefore extremely important to introduce this method as early as possible with constant reference to its application in everyday life. In trying this out in classes I have found that the interest of the pupils is increased, which, in turn, has improved their abilities in the subsequent work. I disagree with the suggestion that "An indirect proof is more abstract than a direct proof. For this reason it is well to choose a sequence of theorems that will allow for a postponement of it until the pupil is well grounded in demonstrations of the direct type";[8] and with the statement that " . . . there is another indirect method that is occasionally needed as a last resort."[9]

The purpose of teaching geometry is not only to familiarize the pupils with the methods of proving geometric facts but also to inculcate in them an appreciation and ability of the "If-Then" kind of thinking which Professor Keyser has so ably propounded. The indirect method as a process of thought is an example *par excellence* of the "If this is so, then that is so" type of thinking and therefore should occupy a prominent place in any teaching of geometry.

Although not to the same extent as the indirect method, the algebraic method has also been slighted to no small degree. Historically there is foundation for this, because algebraic notation and application developed much later than geometric concepts and also because the original Euclid contained no algebra as such. Yet it is a powerful and simple method, the application of which as a tool motivates the study of algebra, at the same time in a large number of cases simplifying the problem. Whenever it is practical and natural to do so, algebra should be employed as an integral part of the work in geometry. There is no doubt that experience has taught us that symbolic expression of any kind facilitates geometric analysis, the expression of the results of the analysis and the manipulation of these results to obtain further relationships. Geometry is replete with possibilities of expressing geometric relations by means of equations. The algebraic method therefore should be given much greater emphasis than it is given at the present time.

3. *Training for citizenship.* Besides the methods discussed above there are many

[6] Upton, C. B. *Ibid.* Page 105.
[7] Upton, C. B. *Ibid.* Page 127.
[8] Hassler and Smith. *Ibid.* Page 356.
[9] Smith and Reeve. *Ibid.* Page 259.

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other processes of thinking that should be used in geometry and which are of considerable importance in everyday life activities. While it is true that there are many more factors attending any life situation than in the realm of geometry and while there is no gainsaying the fact that the facts are not so clear and so well defined, yet the reasoning processes are the same. Therefore the study of geometry is important as a type of training for intelligent citizenry. Such processes are:

1. The distinction between necessary and sufficient conditions. Careful study should be made to develop the idea that a condition may be necessary, sufficient, or both necessary and sufficient; to prove that a condition is both necessary and sufficient, the truth of both the statement and its converse must be established. A simple problem illustrates this notion very clearly:

John said he would go if Henry went.

If we assume the truthfulness of both John and Henry, then Henry's going is a sufficient condition for John's going. Yet, taking the statement at its face value, Henry's going is not a necessary condition for John's going, for we cannot tell what John would do if Henry did not go.

2. Generalization and the need of avoiding hasty generalizations—reasoning from insufficient data.

Guiding the efforts of the pupils in generalization, that is, the discovery of a general statement or principle from a number of particular problems, is a very important phase of mathematical thinking. The inculcation of that attitude of mind which leads a pupil to the discovery of general laws from an array of special cases demonstrates that thinking is more than the mere acquisition of related or unrelated facts.

Modern psychology shouts the praises of this process of generalizing information and the pitfalls that one must guard against in avoiding hasty generalizations. The study of geometry lends itself peculiarly to the proper establishment of this attitude. Groups of related theorems as special cases may be later combined into a generalized statement which includes all the other special cases. Carefully graded exercises may be developed into a generalization, while on the other hand generalization may be shown to be lacking due to insufficiency of data.

3. Fallacies common in everyday reading, speaking, lecturing, and debating.

a. Begging the question.

To argue that this criminal should be punished, because all criminals are a menace to society, begs the question if it has not been shown that this man is a criminal. The use of a name or epithet may lead to fallacious conclusions and very frequently the epithet is more dangerous than the implied proposition would be, for we are less likely to notice that something has been assumed when the proposition is only suggested.

b. Circular reasoning.

This frequently occurs in the geometry classroom and at times is difficult for the pupil to understand. In everyday life the identity in meaning of the two statements in easily seen is short statements, but in a long argument detection is difficult.

c. *Non sequitur.*

In this type of fallacious reasoning the premises may be clear and true but the conclusion does not follow, because it does not belong to the propositions upon which it is based:

(1) *My opponent presents a formidable array of statistics to prove that the country is financially unfit for war; to which I am proud to reply that the old flag has never touched the ground.*

(2) *These lines are parallel because the triangles are congruent.*

4. Reasoning by analogy.

Whatever the subject as the medium of operations may be, I think all are agreed that a pupil's reasoning and good judgment reacting to any given set of conditions in the schoolroom or in everyday

life activity are much more important than the possession of vast stores of information or high degrees of skill in specialized branches.

The study of geometry lends itself very favorably to the inculcation of the attitude of suspended judgment, reflective thought, and logical reasoning. Yet (considering the content materials and the present methods of teaching) how much is transferred over to everyday life situations?

Are not people in general, frequently intelligent, at the mercy of the wiles, tricks, and psychology of the politician and advertiser? This is particularly true when reasoning by analogy is involved. Wherever we go we are confronted with such types of reasoning as:

a. *This athlete smokes this brand of cigarettes to steady his nerves. Therefore this brand will steady your nerves.*
b. *This movie star uses this brand of cream to keep her skin lovely and smooth. Hence this brand will keep your skin lovely and smooth.*
c. *This medicinal compound cured this person (picture and testimonial shown) of this disease. Hence it will cure you of this disease.*

Hundreds of such examples of fallacious reasoning by analogy evident in our everyday experiences could be cited, and the unfortunate outcome is that the gullible public jump to the conclusions without further mental inquiry.

Yet reasoning by analogy is a very powerful tool in thinking provided certain general rules of identical elements are kept in mind. It is not difficult to bring home to pupils the erroneous conclusion that if two cases, facts, or situations agree in several respects that they must necessarily agree in all respects. Analogy alone is notoriously an unsafe guide without a thorough investigation into the relevant and independent characteristics, and the causes and laws of the issues involved.

If we are to prepare our pupils for life it seems absolutely essential to equip them with the knowledge, at least, of this important fallacy in logical thinking.

In the light of the suggestions outlined above and because the purpose of education today is to prepare the child for life and to train for intelligent citizenship, it must be conceded that the method of the training of the mind through the knowledge of geometric facts and formal proofs, *per se*, is no longer tenable. Rather should this training come through exposure to and mastery of methods and processes of thought and through the inculcation of certain habits, ideals, and powers which are to function in the life of the individual. Applications from the pupils' other subjects and from life situations outside of school of the same methods of procedure as those developed in geometry will increase the interest and worthwhileness of the study of a subject which pupils usually regard as thrust upon them "ex cathedra."

Constant reference should be made to problems outside of the field of mathematics and the manner in which the processes of thought in geometry can be, and are used to solve them.

To help the reader realize the possibilities referred to, I herewith append a few examples of such applications supplied me by many teachers and friends interested in this most important phase of geometry teaching. Some are taken from books as noted:

1. On all very high mountains, there is a line called the timber line above which trees will not grow. Are the following conclusions necessarily true?
   a. Since Mount Blue has no trees on its summit, it extends above the timber line.
   b. Since Mount Blue has trees on its summit, it does not extend above the timber line.
   c. Since Mount White extends above the timber line, it has no trees on its summit.
   d. Since Mount Verde does not extend above the timber line, it has trees on its summit. (McCormack, *Plane Geometry, Revised.*)

2. When the steam was on last night, water leaked from your radiator on the floor. The janitor says that the radiator is all right. Convince him by indirect proof that he is wrong. (McCormack, *Plane Geometry, Revised.*)
3. A very heavy package was stolen from John's room. From the last time it was seen there until it was missed, it is known that only Peter and the crippled Henry had been there. You are the prosecuting attorney. By an indirect proof, convince the jury that Peter is guilty. (McCormack.)
4. The clock you sold yesterday has stopped. The customer returns it saying that her maid wound it last night but it will not run. You find that the spring is entirely unwound so you wind it and it runs. The spring would be unwound if the clock had run down or if the spring had broken. Convince the lady that her maid forgot to wind the clock. (McCormack.)
5. You are the doctor. The child has a fever, rash, sore throat, and his tongue is red. He was vaccinated last year. In measles there is a fever, rash, tongue coated white, no sore throat. In smallpox there is fever and rash but the disease is prevented by vaccination.
   In scarlet fever there is fever, rash, tongue red, and sore throat.
   In other diseases there is no rash.
   By the indirect method, diagnose the case. (McCormack.)
6. Mr. Brown spends part of his salary for necessities and the rest of it for luxuries. Mr. Rich, whose salary is twice Mr. Brown's, spends the same amount for necessities as Mr. Brown does, and spends the rest of his salary for luxuries. If the income tax ought to tax the part spent on luxuries, should Mr. Rich pay twice, more than twice or less than twice as much tax as Mr. Brown? (McCormack.)
7. Assume that reducing the gold content of the dollar will ultimately cause all wages and all costs of commodities and property to become double what they were before. Explain whether such reduction is an advantage or disadvantage to:
   a. The man who spends all of his wages each week;
   b. The man who owns a house on which there is a mortgage;
   c. The man who holds the mortgage.
8. Assume that whenever wages are raised, prices are raised too. Does it necessarily follow that:
   a. If wages are not raised, prices are not raised?
   b. If prices are not raised, wages are not raised?
   c. If wages are raised, prices are raised?
   d. If prices are raised, wages are raised? (McCormack.)
9. Assuming that it rains whenever the wind is in the east, which of the following statements are necessarily true:
   a. Since it is raining, the wind is in the east.
   b. Since the wind is in the east, it is raining.
   c. Since it is not raining, the wind is not in the east.
   d. Since the wind is not in the east, it is not raining. (McCormack.)

After each conclusion given below write "Yes" if it is true, "No" if it is false, and "Doubtful" if it may be either true or false.

10. John is absent from school whenever it rains. John was absent on Monday. Therefore it rained on Monday.
11. Mary is absent from school only when she is ill. Mary was absent on Tuesday. Therefore she was ill on Tuesday.
12. a. Smith, Jones, and Brown are candidates for the same office. Smith proves conclusively that Jones can not perform the duties of the position and that he can. Therefore Smith should be elected.
    b. If the answer is not "yes" what additional argument must be given so that the answer will be "yes"?

13. Henry is taller than John; John is taller than Mary; therefore Henry is taller than Mary.
14. Oscar is taller than John; Oscar is also taller than Mary; therefore John is taller than Mary.
15. James agreed to go only if Henry went. James went; therefore Henry must have gone.
16. Mary said that she would go if her mother went. Mary went; therefore her mother went.
17. Under the NRA all Patterson Textile workers receive at least $12.50 weekly. Tom Smith is a Patterson Textile worker. Therefore he receives at least $12.50 weekly.
18. James Brown lives in Patterson and earns $13 a week. Therefore James Brown is a Textile worker.
19. When the drain pipe is stuffed, water does not flow. The drain pipe is not stuffed. Therefore water is flowing.
20. In blood tests, all people are found to be in groups classified as A, B, O, or a mixed group AB. Medical science has proved that if both parents are in the A classification, their children will be in the A group or the O group, but cannot be in the B group. Mr. and Mrs. Smith are both in group A. After their death a person claims to be their son and therefore entitled to the inheritance. His blood is tested and found to be in group B. Is this man the son of Mr. and Mrs. Smith?
21. In order to be graduated from High School a pupil must have (besides other requirements) a 3 unit group in either a foreign language, a science, or mathematics. James Brown was graduated from High School. He did not have a 3 unit group in science. Therefore he had a 3 unit group in mathematics.

    What additional fact must be stated before the answer to the above can be yes?
22. Jones has been successful in every enterprise he has undertaken. Therefore he will be successful in the next enterprise he undertakes.
23. Smith never fails to wear his rubbers when it is stormy. Smith did not wear his rubbers on Wednesday. Therefore it was not stormy on Wednesday.
24. An advertisement circulated by the manufacturer of tooth paste asserts that chemical A is best for cleaning teeth. Tooth paste M contains chemical A. Therefore tooth paste M is best for cleaning teeth. If your answer is not "yes" what investigation is necessary to establish the truth of the third statement given above?
25. Medical science has proved that if a person has once had Yellow Fever and has been cured, that person can never contract this disease again. James Gray had Yellow Fever three years ago. Therefore James Gray is immune to Yellow Fever.
26. All the houses on a certain block are private houses. I live in a private house. Therefore I live on that block.
27. All the houses on a certain block are private houses. I live on that block. Therefore I live in a private house.
28. Every day A goes to school. He rides on a New Lots train. A went to school today. Therefore A rode on a New Lots train today.
29. Every day B goes to school. He rides on a New Lots train. B rode on a New Lots train today. Therefore B went to school today.
30. Only a citizen can be a voter. Smith is a citizen. Therefore Smith is a voter.
31. Jones is a voter. Therefore Jones is a citizen.
32. Whenever a person makes a statement that is not true, he is either ignorant of the facts or he is lying. John told an untruth. He was not ignorant of the facts. Therefore John was lying.

After each statement given below write either "a", "b" or "c" to indicate the correct conclusion

33. Since the NRA went into effect, business has improved. Therefore:

a. The NRA is responsible for the improvement.
b. The NRA had nothing to do with it.
c. More information is needed before a decision can be made.

34. Smith is accused of killing Jones. Five people saw Smith commit the crime. Smith says he can bring fifty people who did not see him commit the crime.
 a. Smith is guilty.
 b. Smith is innocent.
 c. We can't tell.
35. Whenever a baby is hungry it cries. Mrs. Fane's baby is crying. Therefore Mrs. Fane's baby is hungry.
 a. The last statement is true.
 b. The last statement is false.
 c. The last statement is doubtful.
36. Cutting tools have edges and places for handles. These flints have edges and places for handles; they are, therefore, cutting tools.
37. The U. S. is a republic and its citizens are prosperous and contented: we may therefore infer that if Cuba were a republic, her citizens would be prosperous and happy too.
38. Hydrochloric acid turns blue litmus paper red; sulphuric acid has similar properties, and we may infer that it, too, will turn blue litmus paper red.
39. Cotton is grown in the U. S. in a moist warm climate and a sandy soil; we may infer that Egypt which has these characteristics, will also grow cotton.

In conclusion, I wish to state that for a number of years we have been teaching geometry purely as a tool subject. I believe we have succeeded fairly well in creating power to do originals, provided that they are not too removed from the type discussed in classroom study. But is this power an end in itself?

I propose that less emphasis be placed on the solving of originals and that more attention be given to inculcating appreciation of the nature of reasoning, the types of reasoning, types of error in reasoning, their causes, and cure.

If the study of geometry is to be a study of reasoning, then the content and methods must be modified to achieve this aim. The above suggestions may lead the way to make the teaching of geometry more vital and more interesting.

What Day of the Week Was It?

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To find the day of the week on which any given date fell, add four easily found numbers, divide by seven and the remainder is the day of the week desired. With practice the process will take only from five to twenty seconds. For example, for June 5, 1922, we have $1+6+4+5=16=2\cdot 7+2$. The remainder being 2 the day was Monday.

These four numbers we shall call the century, year, month, and day index numbers. The method by which these numbers are determined is described below. A little reflection will convince anyone that to find the day of the week upon which any date falls, according to our present Gregorian calendar, it is only necessary to know the day on which the year one began, according to our calendar, and to know the number of days in excess of an integral number of weeks which elapsed from the beginning of the year one until the date in question. For dates in history under the Julian calendar the proper adjustment must then be made.

The year one began on Monday. If our present calendar be pushed back to that time, Jan. 4, 1, was then the fifth day of the week rather than the fourth. This difference of one we must keep in mind as one of our important numbers to be referred to later. Though actually a part of the century index numbers this one is for convenience combined later with the year index numbers.

We now calculate the century index numbers. The index number for any given century is the number of days in excess of an integral number of weeks in the preceding centuries. We thus have zero as the index number for the first century. This first century, according to our plan for counting leap years contained 76 ordinary years and 24 leap years. Each ordinary year had one day over 52 weeks and each leap year had two days extra, a total of 124 days or 17 weeks 5 days. Hence the index number for the second century is 5. Likewise the second century had five days over n weeks. $5+5=7+3$ so the index number for the third century is 3. The third century adds 5 to this and $3+5=8=7+1$ so 1 is the index number for the fourth century. Since the year 400 was leap year, whereas 100, 200, 300 were not, the fourth century adds 6 days instead of 5, then $1+6=7$, one week and no days so the index number for the fifth century is zero and the cycle of century index numbers runs 0, 5, 3, 1; 0, 5, 3, 1; ——. Hence the index number for the 17th century is 0, 18th is 5, 19th is 3, 20th is 1. These numbers of course are to be remembered, not calculated each time our rule is used.

To find the year index number we need the number of days in excess of an integral number of weeks which have elapsed in the century in question but in the years of that century preceding the year in question. Let n be the number of the year within the century. For 1935, $n=35$. For 1900, $n=100$. Let $\left[\frac{n}{4}\right]$ be the greatest integer in $\frac{n}{4}\cdot$ That is, for $n=35$, $\left[\frac{35}{4}\right]=8$. Now in any century and preceding a given year there have been $\left[\frac{n-1}{4}\right]$ leap years, hence the number of days in excess of an integral number of weeks is $n-1+\left[\frac{n-1}{4}\right]\cdot$ If less than 7 this sum is the year index number. If this sum is greater than 7 divide the

sum by 7 and the remainder is the year index number sought. It is convenient to combine with this year number the one previously mentioned which we have, due to the fact that the year one began on the second day of the week rather than the first. We thus eliminate the necessity of holding this one in mind and at the same time simplify our year number as it is now

$n+\left[\frac{n}{4}\right]$ rather than $n-1+\left[\frac{n-1}{4}\right]$. Thus

for 1935 the year number is $35+\left[\frac{35}{4}\right]=43=$

$6.7+1$ or simply 1. If the year in question is divisible by 100 but not by 400 it is not a leap year and one must be subtracted from the year index number as found by above rule. Likewise if the year is a leap year and the month is January or February one must be subtracted from result found above as in each case one too many leap year days have been counted. Thus for the year 1900 the index number is

$$100+\left[\frac{100}{4}\right]=125=7\cdot 17+6 \text{ and } 6-1=5$$

so the number is 5. Likewise for a date such as Feb. 8, 1932 the year number is

$32+\left[\frac{32}{4}\right]=40=5\cdot 7+5$ and $5-1=4$, the

year number. The year index number will usually have to be calculated each time the rule is applied.

The month index number is the number of days in excess of an integral number of weeks in the preceding months of the year. These numbers are 0, 3, 3, 6, 1, 4, 6, 2, 5, 0, 3, 5, and should be memorized, along with the month to which each corresponds.

The day index number is simply the day of the month in question.

The day of the week for any date may now be calculated. The procedure is as follows. Select the century number 0, 5, 3, or 1. Find the year number, which is the remainder when $n+\left[\frac{n}{4}\right]$ is divided by 7,

with one subtracted if the year is divisible by 100 but not by 400 or if the year is a leap year and the month is January or February. Add the century, year, month, and day index numbers, divide by 7 and the remainder is the day of the week sought. For example.

April 16, 1912
$1+1+6+16=24=3\cdot 7+3$ The day was Tuesday.

February 6, 1912
$1+0+3+6=10=7+3$ Tuesday again.

October 10, 1900
$3+5+0+10=18=2\cdot 7+4$ The day was Wednesday.

January 1, 2000
$1+5+0+1=7=1\cdot 7+0$ The day will be Saturday.

Notice that 0 and 7 are equivalent, each representing the seventh day or Saturday.

Our rule for determining leap year, the latter part of which is not generally known is as follows. A year is a leap year if its number is divisible by 4 but not by 100. If divisible by 100 it is not leap year unless divisible by 400, in which case it is, *unless divisible by 4000*, in which case it is not.

This entire procedure for determining the day of the week may be put into an interesting formula. The formula looks rather forbidding at first but for any given date a large portion of it becomes zero and with a little practice it can be used very rapidly.

C the number of the century, as 20th.

y the number of the year in the century, as 35th.

m the number of the month in the year.

d the day of the month.

N the year, as 1894.

$\left[\frac{n}{p}\right]$ the greatest integer not greater than

$\frac{n}{p}$, or simply the quotient when n is divided by p, neglecting the remainder.

The day of the week is then $K-7\left[\frac{K}{7}\right]$

or, the remainder when K is divided by 7, where K is the sum of the century, year, month, and day, index numbers as given by the formula.

$$K=\left(2\left\{4\left(\left[\frac{c-1}{4}\right]+1\right)-c\right\}+1\right)+\left(y+\left[\frac{y}{4}\right]-\lim_{n\to\infty}\left\{\left(\cos^2\frac{N\pi}{100}\right)^n\right.\right.$$
$$\left.-\left(\cos^2\frac{N\pi}{400}\right)^n\right\}-\lim_{n\to\infty}\left\{\frac{1}{2^{n(m-1)^2(m-2)^2}}\right\}\left\{\left(\cos^2\frac{N\pi}{4}\right)^2-\left(\cos^2\frac{N\pi}{100}\right)^n\right.$$
$$\left.\left.+\left(\cos^2\frac{N\pi}{400}\right)^n\right\}\right)+\lim_{n\to\infty}\left\{\frac{3}{2^{n(m-2)^2(m-3)^2(m-11)^2}}+\frac{6}{2^{n(m-4)^2(m-7)^2}}\right.$$
$$\left.+\frac{1}{2^{n(m-5)^2}}+\frac{4}{2^{n(m-6)^2}}+\frac{2}{2^{n(m-8)^2}}+\frac{5}{2^{n(m-9)^2(m-12)^2}}\right\}+d.$$

The formula as written is good until the year 4000. An additional term is necessary to make it applicable after that date.

It will be noticed that the term $\left(\cos^2\frac{N\pi}{4}\right)^n$ equals 1 or 0 according as the year N is divisible by 4 or not, likewise $\left(\cos^2\frac{N\pi}{100}\right)^n$ and $\left(\cos^2\frac{N\pi}{400}\right)^n$ are always equal to 1 or 0.

Applying the formula to the date September 5, 1923 the first parenthesis gives us the century number 1. The second gives the year number 28. The next term vanishes except for the last fraction in the brace which equals 5. Notice that this term always reduces to the numerator of the fraction in whose denominator one of the numbers subtracted from m is the number of the month in question. Next d is 5. Then $K=1+28+5+5=39=7\cdot5+4$. Hence the day was the fourth day of the week or Wednesday.

Application of the formula is facilitated if one notices in the beginning that the purpose of the first limit subtracted in the second parenthesis is to subtract 1 if the year is a year such as 1700, 1800 or 1900 which were not leap years and that the purpose of the subtraction of the next limit in the same term is to subtract 1 in case the date is January or February of a leap year.

A Problem Play*

A Musical Mathematics Play In One Act and Three Scenes

By Dena Cohen
School No. 49, 1209 Cathedral St., Baltimore, Maryland

Cast of Characters

Alice—A Junior High School child about 12 years old.
Margery—Her friend, same age.
Dorothy—Another school friend.
Tom, Dick, David—Boys in same class.
King Math of Numberland
Guards in the King's palace.
Numbers from 1 to 0.
Plane and Solid Figures—Sq. Rectangle, Parallelogram, Triangle, Hexagon, Circle, Cube, Octagon, Rectangular Prism, Cone, Cylinder.
Dream Fairy

Scenery

The living room scene may be managed by two plain brown screens. In front of these is a table with a chair on each side. The two girls are working with their books and papers piled around. The door at the side may be the right wing of the stage or another screen may form an entrance-exit there.

For the palace in Numberland, plane figures of harmoniously colored cardboard may be attached to a back drop. The throne is a chair set on a small platform and covered with plain cloth on which has been painted in silver tools used in mathematics—compass, protractor, rule, angles, stars made by triangles, circles, etc.

Around the throne are the two screens turned to the opposite side on which have been fastened linen or cardboard hangings painted with geometric designs like mosaics or tiles in a variety of colors.

The piano which helps the chorus sing in tune and dance in rhythm is off stage or down below if in an auditorium.

Costumes

Alice and Friends—Ordinary school dress.
Dream Fairy—Usual fairy's costume.
Guards—Grey material makes a doublet-like jacket with long sleeves; trousers like shorts—on them are painted the four signs of the fundamental operations, $+ \; - \; \times \; \div$. The head piece is a triangular pyramid of cardboard silvered with a visor effect. Silver cardboard sword. Shoes with long points can be made of grey material.
King—Usual costume of royalty, only instead of black ermine tails, have numbers on white "fur" banding.
Numbers—Clown costumes—black and white with numbers forming a design. Stenciled or printed.
Plane Figures:
Square—Silver band around head with a silvered square front and back. Small silvered squares like coat of mail—front and back on grey material basically like the guards' costume.

* This play was the idea of a 7-B Junior High School class of a socialized mathematics lesson. The children suggested the idea, contributed two of the poems and some few lines of the dialogue, chose some of the tunes, and decided on the type of dances. The play was written by the teacher who was a music supervisor before the depression. Now music and mathematics are the subjects taught by her in the Robert E. Lee Junior High School of Baltimore.

The costumes and head dresses were designed by her and executed with the help of the children of the class. The guard costumes were cut out and sewed by a 9-B class under the direction of their sewing teacher in their regular period, material was bought and paid for by each child.

The King and the Dream Fairy provided their own costumes.

Cardboard and screens were furnished by the school.

Ninth grade boys designed and executed the palace background on cardboard and attached the sections to one side of the screens.

The Art, Music and Domestic Science departments in a large Junior High School could make this a correlated project.

Rectangle—Similar—only rectangles in place of squares.
Triangle—Same idea.
Parallelogram—Same idea.
Circle—Head dress same idea as others.
Hexagon—Huge figure—front and back of blouse.
Octagon—Smaller figures on skirt or trouser legs. Shield if desired in respective shape—spear.

Solid Figures:

Cube—Make a cardboard box to fit head—silvered. Same on a spear. Grey blouse and shorts or skirt.
Rectangular Prism—Get a large box. Cover with silver paper or paint. Use small boy. Make holes for eyes and mouth. Paint face. Let boy wear it over his head.
Cone—One with circular base as head dress painted silver. Tin cones sewed on blouse.
Cylinder—Large Quaker Oats box used for display by grocers—silvered or covered with colored paper as desired. Same method as prism.

A PROBLEM PLAY

ACT 1

SCENE 1

(*Living Room. Alice and Margery are found working at a table with books, papers, and pencils in hand doing homework, especially arithmetic. A chorus is heard off stage singing "I can't do that sum."*)

***** Off stage:

"Once there was a little girl
And her name was Alice.
(*Recitation*) She didn't like arithmetic
So treated it with malice."

ALICE: Impossible! This problem is beyond me.

MARGERY: Oh, the eighth one. Yes, I must confess I can't make heads or tails of it. (*She jumps up.*) I have a book at home which explains every kind of problem. I know just where it is, so I'll run out and get it. (*She leaves.*)

ALICE: Oh, dear, I'm tired and sleepy. (*Yawns and puts her head on the table. Dream Spirit appears, dances about the room and beckons. A group of children come in doing a geometric dance. They are the numbers 1 to 0, dressed in clown costumes. The music for this interlude is "I can't do that sum" from "Babes in Toyland." They sing the song and dance out. Alice sits up and begins work again. The door opens and in comes Margery with some boys and girls.*)

ALICE: Oh, hello, boys, where did you come from?

MARGERY: I couldn't find the book anywhere but I thought maybe the boys could help us solve the problem. Come around the table. Alice, you read that problem.

ALICE: Did any of you boys get it to come out right? I mean, to prove?

DICK: No, we thought that you could help us because we were stumped.

ALICE: All right—here goes. If $\frac{2}{3}$ of a number is 128, what is $\frac{5}{6}$ of the number?

DOT: Oh, I remember. You take $\frac{1}{2}$ of 128 and multiply by 3 and then take $\frac{5}{6}$ of the answer.

DAVID: You're off. What's $\frac{1}{2}$ got to do with $\frac{2}{3}$ and what has 5/to do with 6ths?

DICK: Well, how about $\frac{5}{6}$ of 128?

TOM: Oh, yea! 128 isn't the number. They want to know the real number.

DOT: You're all wrong. It's as plain as A B C. You find the number, then $\frac{5}{6}$ and there you are.

KEN: You're not right.

DOT: I am right. I'll bet on it.

TOM: Well, we can try it. If it proves, it has to be right.

ALICE: O.K. Let's see who can get it and prove it first. (*They gather around the table and begin work. A bell is heard outside.*)

TOM: I bet another one of the gang is stuck.

DICK: Guess it must be Arty. (*Alice goes to the door. She returns much excited.*)

ALICE: Oh, the queerest group is outside. I'm afraid to let them in. Oh, here they are. (*Four guards dressed in grey jackets and silver head dresses on which are painted the four signs—addition, subtraction, multiplication and division.*)

CHO. OF CHILDREN: Well, who do you think you are?

GUARDS: We are King Math's men.

CHO.: Ho, ho, ho. Tell us another.

ALICE: Who is King Math?

GUARD: King Math is King of Numberland.

TOM: And where is Numberland? Anywhere near Cumberland?

CHO.: That's a good one!

GUARD: Numberland is a magic land and never on any map. It's in your head—that is, if you are clever enough.

DOT: How do *we* get there as we don't seem to be so clever?

GUARD: We came to take you there to show you how simple and speedy it is.

TOM: Ladies and Gentlemen—get your magic cloaks. You are about to travel. Here we go a-sailing, a-sailing, a-sailing right up to the moon.

GUARD: But you are going. And in a magic balloon. Come. It is getting late. You will have to hurry for King Math is awaiting your arrival. Attention! (*Guards salute, turn and form in line.*) Follow us, boys and girls and watch your step. (*Group look at each other. Suddenly Alice starts after the guards.*)

ALICE: Who's a good sport? I'm ready to go because I want to know how easy that stuff can be. If he says so, it must be so.

CHO.: This way—clear the way—hail, hail, the gang's all here. (*Tune is played and they sing.*)

"Hail, hail, the gang's all here,
We're going to find our numbers!
We're going to find our numbers!
Hail, hail, the gang's all here!
We're going to Numberland!"

SCENE 2

(*Palace of King Math in Numberland. Walls covered with geometric figures, arithmetical signs, tools used in mathematical work. The group of boys and girls rushes in with hair tousled and much out of breath.*)

ALICE: Whew! I never dreamed that we could go so fast. Look at your hair, Dot! It is almost straight up.

DOT: Look at yours—your pigtails are raveling out.

TOM: Say, look at these walls. See anything familiar? There is an oblique parallelogram. Dick, what's the formula?

DICK: Quit that. I can't remember formulas after going through the air 100 miles a minute.

MARGERY: Look at the different kinds of triangles they use here for a decoration. They are certainly smart. Angles, angles, everywhere and not a curve to slide around.

ALICE: Certainly there are curves—look at the circles on the ceiling. As long as it is King Math's palace, he can't omit anything.

DICK: You're right. If they could omit some things maybe we would not be worrying so every night.

GUARD: You'll never worry again. After you see the parade and learn all the history of math, you'll go home rejoicing and be proud of yourselves. (*A trumpet is heard. Guards turn and salute. The King appears. He seats himself upon his throne and motions for the group to come near him. They bow to him, each in turn, and take a place around his throne.*)

KING MATH: Welcome, boys and girls. We have heard about the dreadful time you are having with some of my subjects, so I have decided to find a way out for you. We thought that if you could meet my subjects, things would become more exciting. Now make yourselves comfortable while I have them introduced to you. (*He claps his hands. The music starts—*

"I can't do that sum" and in march in rhythm all the figures—plane and solid—and the numbers from 1 to 0. Each bows to the group and lines up around the stage. If convenient and there is enough time, these may do some sort of dance or form a tableau.)

(Applause by group as each one marches by.)

KING MATH: Now, boys and girls, you see how alive and peppy these subjects of mine are. Not dead nor dull as you sometimes believe them to be. They are right on the job and never fail anyone. If you are accurate they follow your lead, but if you are not sure of yourself, believe me, they certainly get into a mess, and it needs a magician like me to straighten out matters. There is only one way and fortunately that is my way—simple, accurate and speedy. Just to prove how much we need mathematics, some of my subjects will tell you something about its history. Ching come forward and tell us how the Chinese first counted.

CHO.: Honorable soldier, servant of the king,
Stop, we beg of you, and tell us of Ching.

CHING: "Long, long ago
Turtles he had
So he could see;
He said "a lot"
While there were only three.
Years passed by and little Ching
Took his father's place and was crowned king;
Still they counted two, two twos and one—"a lot";
But what we call it—they could not.

New Chings came and old ones went,
Playing in the forest their time was spent;
Counting finally by tens—
Ten and one, ten and two
Up to ten and ten they thought would do;
So their knowledge increased,
Now you see we owe our counting to those deceased.

KING MATH: Good for you, old Ching. We could never have got along without you. Now Bel-Anam, what have you to tell us?

BEL-ANAM: In days gone by when sheep did stray
To drive them home was part of play;
They counted three and then said "many";
Then three and one, two threes, three threes and one,
Two threes and two; oh, there were plenty.
Three threes, three threes and one,
Three threes and two;
But at that time no word said four.
Soon they saw that toes and fingers
Count to ten and ten; so it lingers
To this day; for now as you count,
By tens and twenties you must mount.

(Bel-Anam bows and retires to the side.)

KING MATH: Thank you, Bel-Anam. Now, Ahmes, step forth and tell us how the first book of mathematics was written.

AHMES: Down in the valley of the Nile
Stood a temple, a tremendous pile,
On the walls were curious signs
Which seemed to most just idle lines.
The temple priest one day did tell
The secret that he knew so well;
The number of days in the king's last war
Just so many and never any more.
Now little Ahmes the priest did seek,

And in a voice both low and meek
Begged to learn to read these signs
Which made about the walls such queer lines.

He brought to him a water plant
To cut in strips and lay aslant;
It made an excellent scroll for him
These most curious signs to limn.

As he grew older he did make, for all
Those who came, after Egypt did fall,
A scroll, that told of all he knew—
And thus, the first book of mathematics grew.

(*Bows and goes to side. Applause. Chorus sings the following to the tune of "Santa Lucia."*)

CHO.: Ahmes, oh, great one,
Teacher of math,
Thou hast taught the world
How to subtract;
It shall always be
Told of one who was bold
For thou didst write about
Mathematics which is old;
Ahmes, thou great one
We'll always have faith
In your discovery bold,
For we'll need math
Till we're old.

KING MATH: So, my dear boys and girls, we see that when the world was young it was necessary to learn numbers for they always answered the question "How many" and "How much." Today I heard of a great discussion that you had about a problem.

ALICE: Oh, yes, none of us was sure of the way to find an answer to our problem, so we shall be delighted to have you tell us.

KING MATH: Wouldn't you rather learn how to find the answer yourself? If I show you the HOWS and WHYS maybe you can find out. Would you like that?

GROUP: Oh, yes.

ALICE: I know the problem. If $\frac{2}{3}$ of a number is 128, what is $\frac{5}{6}$ of the number?

KING: All right. Let's get together on this. First we ought to decide WHAT we want to know; after that it will be easy to find out HOW to work it. In this problem, Alice, what is the most important thing?

ALICE: The number.

KING MATH: Exactly. Now let's see HOW we are going to find that number. Do you know anything about it?

ALICE: Yes, I know $\frac{2}{3}$ of it.

KING MATH: Guards, bring forth the cards. (*Guards bring out two cards. On one larger piece which is divided into 2 equal parts is printed 128 in center and $\frac{1}{3}$ at the top of each section. On opposite side 64 in each center. These cards are held by two guards with the fractional side showing.*) How much in each part, Alice?

ALICE: 64.

KING MATH: How did you get this?

ALICE: I divided.

KING: Right. If you have 64 in one part and 64 in another and you must have 3 parts, what number is in the third part?

ALICE: 64.

KING: Turn the cards please. Look, Alice, we have three sections now, therefore what is the whole number?

ALICE: 192. I see. I multiplied this time.

KING: Splendid. You have just told us HOW to do it. Which operations did you use?

ALICE: First I divided to find one part; then I multiplied to find the whole.

KING: Now for the rest of the problem. If you have found the number what is to be done to find the new part of it?

ALICE: $\frac{5}{6}$ means multiplying the number by the new fraction.

GROUP: Hurrah for Alice! Now we know how to do these problems. Divide, then multiply.

KING: And all other problems. Just find

out the HOW and WHAT. Guards, bring forth the guide posts. (*The guards hold out to the view of the group two cards on which we see:*)

WHAT DOES THE PROBLEM ASK?

HOW ARE YOU GOING TO FIND IT?

(*Alice and her friends sing: "I* CAN *do that sum."*)

"Put down 6 and carry 2,
Gee, but this is easy to do,
We just thought and thought and thought
And we're not so dumb;
We all know what the teacher wants,
We can do that sum."

(*They act this song while singing it and do amusing dancing.*)

(*King Math and his subjects applaud and the King takes Alice by the hand and leads her off the stage. Others follow. Tune played as procession leaves stage.*)

SCENE 3

(*Living Room again. Alice still asleep as Margery comes in.*)

MARGERY: Oh, Alice, wake up. I did not mean to stay so long, but I could not find that book so I talked it over with Dad. He says to find out WHAT is wanted and then HOW to get it. Let's read it again.

ALICE: Oh, I know it from memory. If $\frac{2}{3}$ of the number is 128, what is $\frac{5}{6}$ of the number. It's the number we want first of all. I know now. If I have $\frac{2}{3}$ equalling 128, then I can find $\frac{1}{2}$—

2 parts equal 128
1 part equals 64
3 parts equal 3 times 64 or 192

which is the number itself. It is only division of fractions. Now if we have the whole we can find a new part.

MARGERY: Of course—multiplication of fractions.

ALICE: Now it is easy—first you divided and then you multiplied. Hurrah! (*The two girls get to work as the curtain falls while the tune used before is heard again.*)

OUR conception of the structure of the universe bears all the marks of a transitory structure. It is not possible to predict how long our present views and interpretations will remain unaltered and how soon they will have to be replaced by perhaps very different ones, based on new observational data and new critical insight in their connection with other data. . . . By the use of mathematics, that most nearly perfect and most immaterial tool of the human mind, we try to transcend as much as possible the limitations imposed by our finiteness and materiality, and to penetrate ever nearer to the understanding of the mysterious unity of the Kosmos.—From *Kosmos*, by WILLEM DE SITTER.

THE ART OF TEACHING
A NEW DEPARTMENT

A Plan for Individualized Class and Home Assignments

By J. WHITNEY COLLITON
Central High School, Trenton, New Jersey

SOME typical examples.

1) Geometry. The centers of two circles are 80 inches apart. The radius of one circle is 10 inches and of the other is ( ) inches. Find the length of their common internal tangent. (Nearest hundredth). Formula: $n+14$.
2) Algebra. A merchant sold an article of $( ) and thereby lost on the cost as many per cent as the article cost in dollars. What did the article cost? Formula: $\dfrac{n+40}{4}$.
3) Trigonometry. The three sides of a triangle are $AB=65$, $BC=75$, and $AC=(\ )$. Find the radius of a circle that is tangent to AC and also tangent to BA and BC produced. Formula: $n+19$.
4) Algebra. Solve for x.
$$\frac{x^3+23x+(\ )}{x^2-1}+\frac{x^2+2x+1}{1-x}=2$$
Formula: n.
5) Analysis. If the cost per hour for fuel to run a given steamer varies as the cube of its speed and is $40 for a speed of 10 miles per hour, and if other expenses average $( ) per hour, find the most economical rate to operate the steamer on a 500 mile trip. Formula: $10(n+9)$.
6) Write the equation of the locus of a point that moves so that it is always equidistant from the lines $4x-5y=8$ and $8x-10y=(\ )$. Formula: $n-15$.
7) Find the equations of the three altitudes of the triangle whose vertices are $A(0, 6)$, $B(-1, -4)$ and $C(\ )$.
9) Surveying: Find in acres to the nearest hundredth, the area of the field.
   AB—472.5 ft. bears N( )°46′E
   BC—729.89 ft. bears N36°17′E
   CD—826.47 ft. bears N67°41′E
   DE—410.05 ft. bears S15°24′W
   EA—Find length and bearing of this closing line. Formula: $n+19$.
10) Solid Geometry. Find the area of a spherical polygon on a sphere with a radius of 18 inches if the angles of the polygon are 144°, 62°, ( )° respectively. Formula: $2(n+24)$.

You will notice that each example has a ( ) in it. Each pupil is assigned a different number to put in the parenthesis. Hence no two pupils do exactly the same work and no two arrive at the same result.

The writer has over a thousand examples like these, each worked at least fifty times.

Work is handed in on sheets with the heading.

Name............... Example No.....
Your Number........ Number Used....
Show all work in space below.

Result

These papers are sorted so that the same examples are together and very quickly checked for accuracy. If the teacher cares to do so he can indicate the point at which the pupil made an error and the pupil may be assigned other numbers for further practice or may be asked to correct the work using the same number.

Each pupil in the class is given a number n, and for each example there is a formula by which he arrives at the number he must use in the parenthesis. Thus, if a pupils' number is 15 and the formula for a certain example is $2(n+6)$ he uses 42 in the parenthesis. The pupil is credited only for correct results and receives less credit if he has to hand the example in more than once for correction.

These examples are assigned for home work or class room work and there is but little probability that the work handed in has not been done by the pupil to whom it was assigned. The bright industrious pupil who would be perfectly willing to lend his work for another to copy is not nearly as willing to do the assignment twice—once for himself and once for some one else.

I admit that the labor involved in preparing the sets of answers is considerable, but I consider that the improvement in results and especially the elimination of a situation which is morally disturbing justify the work.

A Message From the New President

When you and I began this work in which we are now engaged, we pledged ourselves consciously or unconsciously, to help those who came within the influence of our classrooms to become the highest type of individuals of which they were capable; to become intelligent citizens. We wished for them a development which would enable them to adjust themselves to the civilization of their time, a development which would make them citizens with an interest in and a knowledge of the forces which shape and control their environment. Mathematics has and is definitely contributing to this environment and so it must assume its fair share of the development of our pupils.

But the burden of the pupil's interest and understanding of mathematics falls on each one of us, on each classroom teacher. As we do our work we realize how much the experience and help of others can mean to us.

In 1920, the National Council of Teachers of Mathematics was formed to help us and to further our mutual interests. Each month the Mathematics Teacher comes to your desk and mine, full of contributions we have found time to make, full of a wealth of material and help. As for our yearbooks, their value is greatest to those who have thumbed them most. That you might have closer, more personal contacts with others engaged in the work you are doing, the Council through its board of directors, has held meetings which have increased in number until we have two or three each year. These are planned so that you may meet other classroom teachers, teachers particularly interested in subject matter, teachers of methods, administrators and supervisors. You may meet them in the halls, at round table discussions, at programs, at luncheons and dinners. Their interests vary in range from the arithmetic of the elementary grades through to the subject matter of college level. The Council also has committees at work on material important to us. They report to us through our periodicals and at our meetings. Whenever possible these meetings are brought to our very doors. Since January 1935 there has been one meeting on the Atlantic coast, another in the middle west, another in the Rocky Mountain region and for the end of June one is being planned on the Pacific coast.

We have had able and competent officers and boards of directors. They have done much for us and carried the work of the Council far. The amount of money at their disposal has been limited. You can count on those who now represent you on the board to work hard and give of their best. Surely, all of us who are teaching mathematics, you and I, are anxious both to give and to receive our share of the good things which the Council provides. We can be counted on to extend their benefits to those teachers of mathematics in this country who have not fully realized these opportunities. Let each one of us do the thing he can do best to advance the work of the Council, for in advancing this work we are helping ourselves to a fuller and more abundant life in the work to which we have pledged ourselves.

Martha Hildebrandt

Maywood, Ill.

Whenever inferences occur logically mathematical thinking occurs.—Edward Kasner.

IN OTHER PERIODICALS

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

Algebra

1. Barker, I. C. *A slide rule for quadratic equations.* School Science and Mathematics. 35: 811–13. November 1935.

The author describes a slide rule which will solve readily all quadratic equations whose coefficient of x^2 is 1, whether the equation is factorable or not. Directions and illustrations for constructing the ruler are given, as well as the theory underlying its operation.

2. Frutchey, F. P. *New test.* Educational Research Bulletin. 14: 154. September 1935.

A brief description of the Iowa Algebra Aptitude Test which was designed "for use in sectioning first year algebra students, or for securing a prognosis of success in the subject prior to study."

3. Lewis, Arthur J. *The solution of algebraic equations by infinite series.* National Mathematics Magazine. 10: 80–95. December 1935.

The purpose of the article is "to outline methods of expressing all the roots of an algebraic equation by infinite series and to formulate conditions of convergence for these series."

4. Read, Cecil B. *The treatment of division by zero.* School Science and Mathematics. 35: 801–02. November 1935.

An appeal for a more rigorous and more meaningful approach to the operation of division by zero. A bibliography is included, consisting of eleven references to books where such a treatment may be found.

5. Starke, Emory P. *The theory of numbers for undergraduates.* National Mathematics Magazine. 10: 53–57. November 1935.

The author gives a detailed outline of a course in the theory of numbers which he has been giving to undergraduates for a number of years "with highly satisfactory results as regards both student interest and mathematical growth." A good bibliography is included.

Arithmetic

1. Barnes, E. P. *Effective pupil guidance.* The Business Education World. 16: 131–34. October 1935.

The author claims that we have failed to utilize the possibilities of business arithmetic in our guidance program, and proceeds to outline a detailed plan for the reorganization of the subject so as to eliminate the commonly found defects.

2. Buswell, G. T. *Selected references on elementary-school instruction: arithmetic.* The Elementary School Journal. 36: 211–13. November 1935.

An annotated bibliography of eleven items selected from the articles and books on arithmetic published during the period from June 1934 to June 1935.

3. Connelly, Russel L. *An arithmetic test.* The Instructor. 45: 58+. November 1935.

The test checks the pupil's ability to use tables of length, weight and area, as well as formulas dealing with the finding of the area and volume of common geometric figures.

4. Hurst, Clara E. *Arithmetic for grades III and IV (form II).* The School. (Toronto, Elementary Edition). 24: 140–45. October 1935.

A detailed exposition of the objectives for the teaching of arithmetic in grades III and IV, and of the methods suitable for their realization. There are many helpful diagrams, charts, questions and problems.

5. Pyle, W. H. *An experimental study of the development of certain aspects of reasoning.* The Journal of Educational Psychology. 26: 539–46. October 1935.

The author reports a study designed to discover by experimentation what type of arithmetical problem can be solved, and what kind of literary material can be interpreted by pupils in various school grades.

6. Richeson, A. W. *Warren Colburn and his influence on arithmetic in the United States.* National Mathematics Magazine. 10: 73–79. December 1935.

An interesting biographical essay of one of the early writers of mathematical text books in the United States. The author emphasizes the importance of Colburn's contribution to the elusive art of text book writing by sketching

him in against the background of his forerunners and contemporaries.

7. Rosenberg, R. Robert. *Teaching business mathematics.* The Business Education World. 16: 181. October 1935.

A discussion of the aims and methods that a teacher of business arithmetic should keep in mind for an effective presentation of his subject.

8. Schlosser, Louise. *Arithmetic in the primary grades.* The Virginia Teacher. 16: 99–100. May 1935.

The writer describes a unit of work for the second grade on "The Grocery Store" to illustrate the possible integration of arithmetic with situations taken from life.

9. Watson, Stanley A. *Arithmetic; junior third to junior fourth (grades V–VIII).* School. (Toronto. Elementary Edition). (a) 24: 117–21. October 1935. (b) 24: 202–09. November 1935.

(a) A careful analysis of the teaching of long division. The writer points out the difficulties, the pitfalls, the common errors as well as the preventive measures and remedial exercises.

(b) In this article the author outlines a scheme for helping pupils to see what is required in the solution of verbal problems. He also indicates some of the difficulties in teaching percentage and mentions the methods he found helpful in overcoming them.

10. Wilson, Guy M. *The challenge of one hundred per cent accuracy in the fundamentals of arithmetic.* Educational Method. 15: 92–6. November 1935.

The main purpose of the article is to explain the feasibility of one hundred per cent program in the fundamentals of arithmetic, its desirability and some details of the plan for securing such results.

11. Wilson, Guy M. *Why children fail in arithmatic.* American Childhood. 21: 18. October 1935.

The author analyzes the factors that cause the failure of children in arithmetic and presents a substitute program of study that will make the dreaded subject more useful and less burdensome.

12. Wyatt, Ruth. *A unit of teaching in mathematics.* Educational Outlook. 9: 244–51. May 1935.

A carefully worked out unit on "The volume of a cylinder" for an 8A class

Calculus

1. Hadamard, J. *La notion de différentielle dans l'enseignement.* The Mathematical Gazette. 19: 341–42. December 1935.

The author quotes, with approval, a remark that Poincaré made to the effect that one often has occasion to think in terms of derivatives but very infrequently in terms of differentials. He argues, therefore, that in our teaching of calculus we should do away with the very complicated and unsatisfactory explanations that have classically been given of the symbol *d*.

2. Larson, Allan W. *How a polar planimeter works.* School Science and Mathematics. 35: 932–41. December 1935.

The author explains the theory and operation of a polar planimeter in ascertaining the area of a bounded plane surface. Diagrams, pictures and a bibliography are included.

Geometry

1. Robinson, R. T. *Theorems on the tetrahedron.* The Mathematical Gazette. 19: 356–64. December 1935.

The author states and proves thirteen theorems on the tetrahedron.

2. Sanger, Ruth B. *Correlating geometry and history.* Progressive Education. 12: 470–72. November 1935.

An interesting record of an experiment in which geometry was integrated with history and the history of art.

Trigonometry

1. Leake, John B. *A device for teaching the operation of the sextant.* School Science and Mathematics. 35: 923–24. December 1935.

With the aid of a diagram the writer describes a device that will facilitate the teaching of the operation of the sextant.

Miscellaneous

1. *All non-mathematicians barred.* School Science and Mathematics. 35: 681. October 1935.

A vigorous editorial rebuking all those educators who would delete the word *non* from the celebrated motto that Plato had inscribed over the entrance to his academy.

2. Brown, Christine A. *Learning to use mathematics.* Junior-Senior High School Clearing House. 10: 26–8. September 1935.

The writer is eager to give the student the advantage of an integrated program without depriving him of daily contact with the teacher who is the specialist in his subject. She describes a method whereby the teacher of mathematics is called in by the teacher of other subjects whenever a project is carried on that requires extensive use of mathematical knowledge or technique.

3. Eells, W. C. *1935 as a centennial year in the history of mathematics.* The American Mathematical Monthly. 42: 171–73. March 1935.

An interesting compilation of the important mathematical events that took place through the centuries in the years ending in 35.

4. Ettlinger, H. J. *Mathematics as an experimental science.* National Mathematics Magazine. 10: 3–8. October 1935.

The author discusses the experimental nature of mathematics with respect to (a) the methods of mathematics, (b) the application of mathematics, (c) the teaching of the subject itself, and (d) the development and growth of the organic body of mathematics.

5. Forsyth, A. R. *Old tripos days at Cambridge.* The Mathematical Gazette. 19: 162–79. July 1935.

Interesting reminiscences of the circumstances of mathematical study at Cambridge (England) between fifty and sixty years ago.

6. Frandsen, Arden. *On mathematics essential for elementary statistics.* School and Society. 42: 263–64. August 24, 1935.

As a result of an experimental study the writer makes the following conclusions:

a. Knowledge of certain phases of elementary mathematics constitutes a significant factor conditioning achievement in elementary statistics.
b. Achievement in statistics can be predicted more effectively with a test of certain fundamental rules of algebra and arithmetic than with a test of general intelligence.
c. The predictive efficiency of a weighted combination of the two tests is superior to either used alone.

7. Freeman, J. B. *The number concept.* The Mathematical Student. (India). 3: 1–8. March 1935.

This is not the place to take issue with the writer on many fundamental questions that he glibly passes over. The following quotation will suffice, however, to indicate his philosophic bias. Mathematics "was just as exact a science a century ago, and also a thousand years ago as it is today. . . . Numbers are not in things; it is our mind that puts them into them. Just as our minds abstracted them from things, so in the same way and exactly by the reverse operation and with the same limitations we can apply them to things."

8. Jeffery, R. L. *Productive scholarship in the undergraduate colleges.* The American Mathematical Monthly. 42: 364–69. June-July 1935.

After a very clear and specific analysis of the problem, the author concludes that "creative scholarly work is one of the obligations, perhaps the main obligation, of the undergraduate teacher."

9. Keyser, Cassius Jackson. *Three great synonyms: relation, transformation, function.* Scripta Mathematica. 3: 301–16. October 1935;

An excellent elucidation of three consepts that loom big in philosophy, mathematics and logic, together with speculative excursions into interesting bypaths.

10. Langer, Rudolph E. *Reflections of a college teacher on mathematics in the high school.* National Mathematics Magazine. 10: 35–43. November 1935.

This article is welcome proof that the teacher of mathematics in the colleges is at last awakening to the realization that he and the teacher of mathematics in the high schools share common interests and face common enemies.

11. Miller, G. A. *Correcting errors in the histories of mathematics.* School Science and Mathematics. 35: 977–83. December 1935.

The author points out and corrects some errors in well known histories of mathematics and in a standard dictionary.

12. *The need for the re-orientation of mathematics in the secondary school—a symposium.* a. Ploenges, E. W. *From the point of view of the colleges.* b. La Rue, J. T. *From the view point of the high school teacher.* c. Hill, W. H. *From the view point of modern educational theory.* Bulletin of the Kansas Association of Mathematics Teachers. 10: 1–9. October 1935.

A lucid and competent discussion on a crucial problem.

13. Nyberg, Joseph A. *Mathematics for the non-collegiate.* School Science and Mathematics. 35: 905–10. December 1935.

A good exposition of the objectives of a course in mathematics for the non-collegiates, and an enumeration of the topics that should be included in such a course. Two text books are mentioned in which such a course is worked out.

14. Schaaf, William L. *Current trends in junior high school mathematics.* School Science and Mathematics. 35: 959–69. December 1935.

The author reports on the answers he received to a questionnaire he sent out in order to

determine the current curricular practices in junior high school mathematics.

15. Sisam, C. H. *Some comments on the secondary mathematics situation.* National Mathematics Magazine. 10: 25–27. October 1935.

The writer suggests that such mathematics as is taught in the first grade be moved to the second, etc. in order to prevent its complete elimination from the curriculum because of its difficulty.

16. Sleight, E. R. *Interesting the superior student.* National Mathematics Magazine. 10: 58–62. November 1935.

The author describes a method he has been employing at Albion College to interest the superior mathematical student in the work of the class room and in the activities outside of it.

17. Sleight, E. R. *Student tutorial system in freshman mathematics in Albion College.* National Mathematics Magazine. 10: 101–03. December 1935.

A description of an interesting system introduced for the purpose of helping those students in freshman mathematics who have difficulties in adjusting themselves to the new environment.

18. Smith, David Eugene. *Changes in elementary mathematical terms in the last three centuries.* Scripta Mathematica. 3: 291–300. October 1935.

A fascinating study of the transformation in meanings that many mathematical terms underwent in the last three centuries.

19. Watson, J. Donald. *Educational procedure in mathematics is misdirected.* School Science and Mathematics. 35: 799–801. November 1935.

The author argues plausibly that "colleges and universities should broaden their entrance requirements relative to mathematics so as to recognize the need of business and social science students for a background of commercial arithmetic and bookkeeping."

20. Wren, F. L. *A survey of research in the teaching of secondary algebra.* Journal of Educational Research. 28: 597–610. April 1935.

The article is a report of a survey of the literature, since 1905, on the research in the teaching of secondary algebra. The three hundred and seventeen studies which were examined are classified into forty-three distinct research problems, which in turn were grouped under eight major topics of vital importance in the teaching of algebra.

Politics and Arithmetic

Congress having overridden the veto of the Bonus Bill by the President, he naturally requests it to provide the money for carrying out the law which it had made. The usual form of a special appropriation bill, such as Mr. Roosevelt asked Congress to pass, is that the money shall be paid out of funds in the Treasury "not otherwise appropriated." But now there are no such funds—at least none anywhere near the $2,400,000,000 required. Where and how to get the cash to pay the ex-soldiers is the new worry thrust upon Congress. It recoils with horror from the thought of levying taxes in order to pay the bill of the veterans. That step must be put off until after the election. Meanwhile, is there no clever device, no juggling with an unsecured currency, no bookkeeping legerdemain by which the Treasury can be put in funds for the purpose desired? This is the question which now has first place in the minds of Congressmen.

It seems to be a contest between political motives and the rules of arithmetic. Many new ideas and methods have come into being within the past three years in the realm of our public finance. When an orator once said that certain things could not be done because the law of supply and demand would prevent it, one of his hearers promptly proposed the repeal of that law. That would be difficult, yet easier than to repeal the laws of addition and subtraction. They were a continual annoyance and grief to Mr. Micawber, though he never imagined that he could get rid of them. Neither can Congress, even if it postpones their application. Ultimately, arithmetic will prove that it is mighty and must prevail. Every dollar spent by the Administration will be, in the end, a dollar taken from the pocket of the taxpayers. It is a harsh truth, and no one can wonder that Congress, being made up of politicians and not mathematicians, is unwilling to face it or act upon it. But the cold and implacable figures will look them straight in the eye both before and after election day.—From *The New York Times*, January 30, 1936.

NEWS NOTES

The following comment from the *Educational Research Bulletin* for November 13, 1935 should be of interest to the readers of *The Mathematics Teacher:*

An Outstanding Yearbook

The Tenth Yearbook of the National Council of Teachers of Mathematics[1] is outstanding. The volume opens with an article by Brownell that should be a part of the curriculum in every teacher-training institution. In this article Brownell considers the three prevalent theories of arithmetic teaching. His criticism is clear, just, and entirely free from the abuse that is sometimes found in such articles. Brueckner follows with a report of an "Analysis of Instructional Practices in Typical Classes in Schools of the United States." Reports were received from 505 classes of Grades IV, V, and VI in large cities scattered from New England to California. His results will be of interest to all who need information concerning the present status of arithmetic teaching and particularly to those teacher-training institutions which wish to adjust their training to prevailing practice. In the next two articles Buckingham and Buswell, respectively, ably reinforce Brownell's argument for more emphasis on the meaning of arithmetic. Hanna with a committee of teachers gives a report of a survey of the opportunities for the "Use of Arithmetic in an Activity Program." This report is unusually impressive and free from the evangelistic tendencies which some progressive educationists manifest in their writing and speaking. The survey covers the activities used in Grades III and VI for a period of four months in six schools including the Horace Mann and Lincoln schools of Columbia University. With commendable honesty the committee concludes that "functional experiences of childhood are alone not adequate to develop arithmetic skill."

Johnson has an excellent article on "Economy in Teaching Arithmetic" which demonstrates again the futility of most of the current emphasis on the teaching of common fractions. The article encourages one to believe that the teaching of arithmetic in this field may yet become an intelligent procedure.

Unfortunately, that hope is considerably chilled when one finds in the next article that teachers of training courses in 129 institutions vote strongly for less emphasis on the objectives of arithmetic and history of arithmetic and more emphasis on drill exercises and common fractions. This article is entitled "Current Practices in Teacher-Training Courses in Arithmetic." It is well done, as are Overman's treatment of "The Problem of Transfer in Arithmetic" and Repp's discussion of "Types of Drill in Arithmetic" which follow.

At this point there is a sort of critical review written by David Eugene Smith. While this article is interesting and instructive, it is difficult to resist the suspicion that the author is facetious at times. How otherwise could one who has written that "not a wheel could turn nor a ship sail the sea without mathematics" be found predicting "that reducing fractions to lowest terms . . . will be among the historical curiosities and its handmaid factoring . . . will be put by its side in the museum." Has not the professor for the moment forgotten the mathematics of the shop, the high school, and the college? It was to be hoped that such narrow interpretations of social utility were, themselves, in the museum by this time. If the common man is now asinine enough to try to get along as far as possible without arithmetic, it surely does not follow that there is no hope for him in the future. Furthermore, it must not be forgotten that the common man, however dumb he may be, is still the original source of all specialists and indeed of all creators of culture.

In the next article " 'The Mathematical Viewpoint' Applied to the Teaching of Elementary School Arithmetic," Thiele presents another strong appeal for the meaning side of arithmetic. Then Wheeler presents a most challenging article on "The New Psychology of Learning." His is the Gestalt point of view. Reading his article leaves the impression that nothing short of an educational revolution will save us.

In the final article, Upton lines up with the automatists, in an article entitled "Making Long Division Automatic." Upton has spared no pains in making this careful and detailed study. He presents strong evidence in favor of the increase-by-one rule. Unfortunately, his statistics leave much to be desired. For example, he reports that the increase-by-one rule should operate in total area of 18,090 examples. The increase-by-one rule succeeds with 14,544 of these,

[1] National Council of Teachers of Mathematics. *Tenth Tearbook: The Teaching of Mathematics.* New York City. Teachers College, Columbia University, 1935. vii+289 pp. Price $1.75 postpaid.

but the conventional rule also succeeds with 8,444. This would seem to give a total of 22,988 instead of 18,090. An unbalanced table is supposed to be a "thorn in the flesh" to a mathematician and statistics of this sort ought not to be found in a mathematical yearbook. The difficulty can be explained, but the explanation comes at the expense of the increase-by-one rule.

The foregoing criticism has had to be sketchy because of space limitations. The Yearbook is exceedingly well done and tremendously heartening to those who hope for better things in arithmetic teaching.—W. J. Osburn.

A Mathematics Conference was held at the Lord Jeffery Inn, Amherst, Massachusetts, beginning with dinner at six-thirty on Wednesday evening, Sept. 4, 1935 and closing in the afternoon of Sept. 11. President King generously placed at the disposal of the conference the facilities of the Amherst library.

The conference was in the nature of a series of round table discussions under the guidance of a group of leaders. The general topic of the entire conference was "A Consideration of the Wisest and Soundest Mathematics Curriculum for Secondary Schools" with special stress on a consideration of materials and methods to enrich mathematics at the secondary school level, opening to students a sense of the perspective of the subject.

The conference was fortunate in securing the services of able and distinguished leaders for the conference. Both the college and the school point of view were represented. The leaders were: Professor Marguerite Lehr of Bryn Mawr College; Professor H. W. Brinkmann of Swarthmore College, formerly of the mathematics department of Harvard University; Professor George H. Mullins of Barnard College; Mr. Joseph Jablonower of the Fieldston School, New York City; Mr. Rolland Smith of the Springfield (Massachusetts) High School.

A definite schedule of lectures and discussions was arranged for the morning and evening sessions. Afternoons were left free for informal discussion by groups with common interest in special problems. For the convenience of teachers interested in the choice of text books, the committee arranged for a collection of sample copies.

Although the conference was planned primarily for the benefit of secondary schools, teachers in the elementary schools here encouraged to come, in order that they might find the content of the conference of value to them as giving perspective to the elementary work.

Miss Lehr presented certain aspects of the higher geometries that have significance for the teacher of elementary geometry; she defined her general topic as "Intuitive and Abstract Approaches to Geometry."

Professor Mullins made available some interesting material especially connected with the use of graphs and other materials enriching the secondary course. This material is partly material derived from continental sources and is not available in English.

Mr. Jablonower is the head of the mathematics section of the Commission on Secondary School Curriculum of the Progressive Education Association. This commission has been at work for two years under the chairmanship of Mr. Vivian T. Thayer. Its preliminary publications will probably be issued next winter. Mr. Jablonower reported the progress of the mathematical section of this commission and also defined for discussion certain questions and moot points yet to be settled.

Mr. Rolland Smith chose as his subject, "The Development of Methods of Teaching Mathematics Which Will Bring Out Meanings." Mr. Smith has been a member of the Commission on Examinations in Mathematics of the College Entrance Examination Board. He is actively interested in secondary school work and for several summers has given courses at Teachers College, Columbia, in the teaching of mathematics.

The Committee on arrangements was as follows: Natalie M. Longfellow, Secretary, The Shipley School, Bryn Mawr, Pennsylvania; Frederick Fraser, The Hill School, Pottstown, Pennsylvania; Ernest E. Rich, Lawrenceville School, Lawrenceville, New Jersey; George R. Wilson, The Taft School, Watertown, Connecticut; Mary Helen Macdonald, The Winsor School, Boston, Massachusetts; Elizabeth Forrest Johnson, Chairman, The Baldwin School, Bryn Mawr, Pennsylvania.

It is hoped that a more complete report of the conference may be given later in *The Mathematics Teacher.*

Significant Facts About American Education

Twenty-three of every 1000 adult Americans are college graduates.

One hundred twenty-five of every 1000 are high-school graduates.

These statements from the Federal Office of Education accompanied many others and the announcement that American Education Week would be observed last year throughout the United States from November 7 to 13. Other pertinent facts on education in the United States reported are:

The chances of a boy or girl going to high school, which were only 1 in 25 in 1890 are now 1 in 2.

The chances of a boy or girl going to college, which were only 1 in 33 in 1900 are now 1 in 6.

One of every 4 Americans attended some kind of school during 1934.

Of every 1000 pupils in fifth grade, 610 enter high school, 260 graduate from high school, 160 enter college and 50 graduate from college.

Ten cents per day paid by every person of voting age in the United States would pay the entire bill for public education: Per year for each child: Elementary, current expense, \$67.82 high school, \$144.03; college and university, \$500.

Costs per school day per child in public elementary school: 39 cents; in high school: 80.9 cents.

Costs per hour per child in public elementary school, 7.8 cents; in high school, 16 cents.

Costs per hour per class (average of 39 elementary pupils) \$3.04; (average of 25 high-school pupils) \$4. Of these costs 75 per cent is for providing instruction by trained teachers and supervisors.

The above facts have been brought together largely from statistics collected on a nation-wide scale by the Office of Education in Washington, D.C.

A new journal called "The Educational Abstracts" was launched last month under the editorship of Norman J. Powell. Helen M. Born, Daniel Green, Sylvia D. Powell, and H. Robert Weiss are Associate Editors. There is also a long list of Cooperating Editors.

Abstracts will be classified under the following headings:

1. Administration and Supervision
2. Adult Education
3. Arts and Crafts
4. Character Education and Behavior Problems
5. Child Development and Parent Education
6. Commercial, Vocational, and Industrial Education
7. Comparative Education
8. Curriculum
9. Education of Exceptional Children
10. Education of the Physically Handicapped
11. Educational History and Biography
12. Educational Psychology
13. Educational Sociology
14. Elementary Education
15. Fine Arts and Music
16. Guidance and Personnel
17. Health and Physical Education
18. Household Arts.
19. Language Arts
20. Mental Hygiene
21. Negro Education
22. Philosophy of Education
23. Preschool and Primary Education
24. Professional Education
25. Religious Education
26. Rural Education
27. Sciences
28. Secondary Education
29. Social Studies
30. Teacher Training
31. Test and Measurement techniques
32. Visual Education

Abstracts will be cross-indexed when the content falls under two or more headings.

The International Congress of Mathematicians will be held at Oslo, Norway from July 13 to 18, 1936.

For the benefit of American mathematicians attending the Congress, it is hoped to arrange one or more Trans-Atlantic houseboat parties which will afford opportunities for fellowship similar to those enjoyed at the summer meetings of the Society and Association. Not all members may find it possible to go on the same sailings, but so far as convenient it will be desirable to concentrate on a few sailings. With this object, the American Mathematical Society has reserved for the use of mathematicians blocks of desirable space in Tourist Class and in Cabin Class on certain ships which seem especially suitable in economy, comfort and convenience. The sailings have been chosen so as to leave soon after the end of the school year and to return just before the Harvard meeting of the American Mathematical Society and the Mathematical Association of America from Aug. 31 to Sept. 5. A passenger can go both ways by these selected sailings or combine one of them with a different sailing in the other direction. In addition to these sailings one can have complete information on the services of all lines. As replies come those in charge hope to be able to satisfy the personal wishes of the mathematicians going to Oslo and, if necessary, to form other parties in order to secure for them the company of some of their fellow mathematicians. By a careful choice of forwarding routes and services every effort will be made to secure the best return possible for the money.

For further information write to Professors R. W. Brink and A. L. Underhill, University of Minnesota, Minneapolis, Minn.

Clarsen, Oklahoma High School has 100% membership in the National Council of Teachers of Mathematics. The teachers are Martha Denny, Marge R. Stevens, Tom Mallory, Julia McDenny, Mildred McCorkle, Undine Butler, Grace Dupree and Bernice Gordon. This is the spirit that will put mathematics and the National Council on the map.

Tulsa, Oklahoma also has 33 members (100%) in the National Council of Teachers, of Mathematics and last week at the St. Louis meeting applied for a Charter to become a branch of the National Council.

Such cooperation is what we need now more than anything else.

The Association of Teachers of Mathematics in New England held its 33rd Annual Meeting at Boston University in Boston, Mass. on Saturday Dec. 7, 1935.

Morning Program

10:15: Social Period.
10:25: Business Meeting.
10:30: Speaker: Dr. William D. Reeve, Teachers College, Columbia University.
Topic: "A Four Year Curriculum in High-School Mathematics, with Particular Reference to the New C. E. E. B. Requirements."
Dr. Reeve's talk was followed by a general discussion of all phases of the new Alpha, Beta and Gamma examinations.

Afternoon Program

2:00: Speaker: Miss Helen G. Murray, Oliver Ames High School, North Easton.
Topic: "Diophantine Analysis."
3:00: Speaker: Prof. C. H. W. Sedgewick, Connecticut State College.
Topic: "Mathematical Stimulants."

The dates for the meetings for the remainder of the school year follow:

March 7. Midwinter Meeting, place to be announced.

April 4. 7:00 P.M. Cambridge.

May 2. 10:15 A.M. and 2:00 P.M., Boston.

Council for 1935

Ray D. Farnsworth, President, Chauncy Hall School.

Professor Elmer B. Mode, Vice-President, Boston University.

Harry D. Gaylord, Secretary, Browne & Nichols School, Cambridge.

Harold B. Garland, Treasurer, High School of Commerce, Boston.

Miss Margaret E. Lougee, F. A. Day, Junior High School, Newtonville, Mass.

Professor Wm. Fitch Cheney, Jr., Connecticut State College.

Professor Harley R. Willard, University of Maine.

Miss Sarah J. Bullock, Arlington High School.

Dr. Helen G. Russell, Wellesley College.

Preston W. Smith, Rivers School, Brookline, Mass.

Past Presidents

*Edgar H. Nichols, '03; William A. Francis, '05; *Charles D. Meserve, '07; Charles A. Hobbs, '09; Archibald Galbraith, '11; *William B. Carpenter, '13; Julian L. Coolidge, '15; Harry B. Marsh, '17; William R. Ransom, '19; Walter F. Downey, '21; A. Harry Wheeler, '23; Lennie P. Copeland, '25; Wm. L. Vosburgh, '27; Raymond K. Morley, '29; Alexander C. Ewen, '31; and Ralph Beatley, '32.

* Deceased

The Mathematics Section of the New Mexico Educational Association met at Albuquerque, New Mexico, October 31, November 1, 1935. Chairman, Ernest L. Harp, Jr., Roswell, New Mexico. The program follows:

October 31

"Promoting the Interest of Jr. High School Pupils in Mathematics," Evert A. Snyder, Gallup.

"Some Applications of Business Mathematics," Dr. H. D. Larsen, University of New Mexico.

"Mathematics for Physics Pupils," Roy Rice, Tularosa.

"Some Suggestions for a Mathematics Club," Miss Olive Whitehill, Deming.

November 1

"Magic Squares," J. E. Gilbert, University of New Mexico.

"Some Mathematical Revisions," Dr. Paul K. Rees, New Mexico College of A. and M. A.

"Mathematics in the Integrated Curriculum," Miss D. Higbee, Springer.

"Mathematics and Pupils," Miss Glady Palmer, Roswell.

"Development of Modern Methods of Calculation," Terrill Haymes, Jr., Eastern New Mexico Jr. College.

Business Session at which the following officers were elected: President, Ernest L. Harp, Jr.; Vice-President, Miss Olive Whitehill; Secretary-Treasurer, Evert A. Snyder.

At the annual luncheon Dr. C. V. Newson of the University of New Mexico gave an interesting talk on "Mathematical Cranks."

The Tenth Annual Conference of Teachers of Mathematics in cooperation with the University of Iowa Extension Division was held on Oct. 11 and 12, 1935 at Iowa City, Ia.

The following program was given:

Friday Morning, October 11, 1935

North Room, Old Capital
Roscoe Woods, Presiding

10:30 A.M. Address: "The New Psychology of Learning with Particular Application to Mathematics," William Betz, Rochester, New York.

10:45 A.M. Address: "Bridging the Gap in Mathematics Between High School and College." A. R. Congdon, Lincoln, Nebraska.

Discussion

Friday Afternoon, October 11, 1935

North Room, Old Capitol
Ruth Lane, Presiding

1:30 P.M. Address: "The Meanings and Uses of Certain Averages." H. L. Rietz, University of Iowa.

2:00 P.M. "Helpful Devices and Methods," Jo Brown, Clinton, Iowa; Myra Newsom, Chariton, Iowa; Arnold Jones, Belle Plaine, Iowa; Ida McKee, Newton, Iowa; Vernon Price, Iowa City, Iowa; Rose Miller, Ottumwa, Iowa.

Followed by contributions from members of the audience.

Friday Evening, October 11, 1935

Iowa Memorial Union

6:00 P.M. Conference Dinner, L. E. Ward, toastmaster.

Informal discussion of present problems regarding mathematics requirements in high school.

Saturday Morning, October 12, 1935

North Room, Old Capitol
N. B. Conkwright, Presiding

9:30 A.M. Address: "Do the Changes in Secondary School Mathematics in the Changing World Indicate Progress?" A. R. Congdon.

10:00 A.M. Address: "Temporary and Permanent Outcomes of Instruction in Ninth Year Algebra." E. F. Lindquist, University of Iowa.

10:30 A.M. Address: "The Study of Mathematics as an Essential Element in Secondary Education." William Betz.

The Mathematics Section of the Southwestern Ohio Teachers Association held its Annual meeting at Cincinnati on Oct. 20th, 1935 in McMicken Hall at the University of Cincinnati.

Mr. Calvin W. Young of Amanda, Ohio was chairman and Mr. Gordon Tyler of Amanda was secretary. The program follows:

Election of Officers.

Address—Ray G. Wood, Director Ohio Scholarship Tests, State Department of Education.

Address—H. C. Christofferson, Chairman Mathematics Committee, Every Pupil Tests, Miami University.

At the St. Louis Meeting of the National Council of Teachers of Mathematics Miss Martha Hildebrandt of Maywood, Ill. was elected president. Miss Mary Kelly of Wichita, Kans. was elected second vice-president. The three new members of the board of directors elected at the St. Louis meeting are Dr. E. R. Breslich of Chicago, Ill. Professor Virgil Mallory of Montclair, N. J. and L. D. Haertter of Clayton, Mo.

Professor H. E. Slaught of Chicago, Ill. was unanimously elected honorary president of the National Council of Teachers of Mathematics and Professor W. S. Schlauch was chosen to take Professor Slaught's place as Associate Editor of THE MATHEMATICS TEACHER.

Section 19 (Mathematics) of the New York Society for the Experimental Study of Education has held three meetings this year. At the first meeting Professor W. D. Reeve, editor of THE MATHEMATICS TEACHER spoke on "Present Criticisms of Mathematics and What to do About Them."

At the second meeting Mr. R. R. Smith of Springfield, Mass. spoke on "An Experiment in Teaching the First Six Weeks of Demonstrative Geometry."

At the third meeting Professor Harold Hotelling of Columbia University gave a very illuminating lecture on "Little Known Applications of Mathematics." It is planned to publish Professor Hotelling's paper later in THE MATHEMATICS TEACHER or in one of the National Council Yearbooks.

◆ NEW BOOKS ◆

Intermediate Algebra. By R. W. Brink. D. Appleton-Century Company, 1935, XII+268. Price \$1.35.

This book is designed for use in college classes in which the students have previously had only two semesters of high school algebra and in secondary school classes for students who are preparing themselves for college. In the case of college and preparatory students who have had only two semesters of algebra the book is intended to supply such students not only with the necessary preparation in algebra but also for courses in physics, statistics, biology, business, medical science, and other sciences for which strictly college algebra is not required.

The idea of the function is stressed as it should be and its early introduction is a feature. The book is adaptable to courses of varying lengths and purposes and should be of interest to all teachers of algebra.

Mathematics at Work. By Geo. H. Van Tuyl. American Book Company, 1935, X+454. Price, \$1.

This book is intended to meet the needs of those students who prefer a less formal and more cultural course in mathematics than has been given in most of the traditional texts. Emphasis on the fundamentals of arithmetic is combined with a general, or cultural, study of algebra, geometry, and trigonometry.

The text is divided into two parts, each of which contains a half year's work. The first part is in lesson units each stressing one topic, with review and drill on topics of previous lessons. Diagnostic texts and remedial drills appear throughout the first part.

Part II is intended to develop accuracy and skill in handling the fundamental operations of arithmetic. It supplies further problems to which the operations of Part I may be applied and furnishes the content material for a cultural study of algebra, geometry, and trigonometry.

Altes und Neues vom Kreis. Dr. W. Lietzmann. Leipzig, Teubner, 1935. Pp. IV+47. Price, 1.20 RM.

This booklet is one of a series (Reihe I) in the well-known Mathematisch-Physikalische Bibliothek, written by scholars of recognized standing, and sold at such a nominal price as to place each volume within the easy reach of all teachers of mathematics. For many years this reviewer has expressed the hope that America might sometime produce books of this kind, as France, Italy, England, and Germany have done. We have the scholars to prepare the texts, we can print about as cheaply as other nations, but the books are not forthcoming in America to any great extent. Is the difficulty due to the lack of interest on the part of teachers? If so, how can this interest be aroused? Our country publishes a seemingly endless number of books on education, generally of a very mediocre type, but we do only a little, now and then, to encourage an increase of interest in and knowledge of the most delightful parts of mathematics.

The fact that this little book is written by Dr. Lietzmann, a man well known in this country and an "Honorarprofessor" in the University of Göttingen, assures its scholarly nature and its simplicity and clarity of statement.

It might seem at first thought that the title of *Altes und Neues vom Kreis* suggests a kind of impossibility. We tend to think that, for our secondary schools, there is plenty that is old in connection with the circle, but that nothing new has developed since Euclid's day. Dr. Lietzmann at once dispels the doubt as to the existence of both the new and the interesting as concerns this familiar geometric form.

He begins with an introduction, largely historical, and then discusses the definition of a circle and its parts and calls attention to its properties. The teacher who thinks that he knows all this will find that he has still a number of truths to learn. For example, we all know that a circle is a closed figure of constant width, but is it the only closed figure having this property? If not, is there more than one? A circle has axial and central symmetry, and so has a regular inscribed polygon; of what other kinds of figures connected with the circle can this be said? Dr. Lietzmann discusses this question and others of similar nature concerning tangents, concentric circles, and the relation of angles and chords not commonly found in school textbooks. Some interesting relations of inscribed and circumscribed figures are also given, not all of them generally found in our texts. There is likewise an interesting discussion of elementary methods of measuring the circumference and finding the area of a circle. The book closes with a brief but worth-while treatment of the Lunes of Hippocrates.

Although our best textbooks in geometry touch upon all these features, the teacher will probably find much that is new to him, and will be glad to have it at hand for his class work.

David Eugene Smith

Integrated Mathematics. Book I. By Dr. John A. Swenson. Ann Arbor, 1935; VI+466. Price $1.47, subject to discount.

This is one of the most destructive books on elementary algebra that has appeared in this country—destructive of useless material, destructive of mere tradition, destructive of courses of study made by teachers without vision, and destructive of much in the way of outmoded requirements for admission to college. Dr. Swenson is a "voortrekker" as they say in the Transvaal—an explorer, a pioneer, a crusader, and a fighter for what he believes to be right. If he has to storm an antiquated castle of tradition, with its towers of prejudice, its stagnant moat, and its unstable drawbridge, he does not join the group of educators who throw puff-balls over the walls, but he brings his big guns to bear and his airplanes to attack from above. His is not a venture like the Children's Crusade of the Middle Ages—great noise followed by a hopeless failure; it is a man's battle against stagnant minds, and no quarter is asked or given.

Leaving the historical allusion, the purpose of which was to say that here is a book that attacks tradition, which any general educator can do, usually with no constructive ideas, let us see what Dr. Swenson is attempting to do. In the first place he lays aside all traditional definitions. In a conversational way he tells what certain words mean, but he does not define. He knows full well that neither he nor any other teacher can define "mathematics," or "algebra," or "unit," or any other words which the earlier writers attempted to make clear to pupils by making them obscure through much verbiage. Next, he does not raise unnecessary barriers to separate arithmetic, algebra, geometry, trigonometry, and (later) the calculus. On the other hand he does not fail to recognize the fact that there is a definite dividing line between the mechanical solution of equations and the logical analysis of a solution in geometry. The work is divided into chapters, their sequence being determined by the use which each makes of the preceding one. Thus geometric problems follow equations, apparently because the latter are needed in the simple problems by which the former introduces the pupil to the reasoning demanded in the approach to geometric proof.

In the course of the work of a school year the pupil has an introduction to the Cartesian coordinate system in the plane, chiefly for the purpose of the study of graphs and a clearer understanding of irrationals, approximations, and mensuration. The work in mensuration leads to a brief treatment of trigonometry, which in turn leads to a chapter on quadratics.

On the whole Dr. Swenson has produced a book with which every teacher of high-school mathematics should be familiar, and upon which he should draw for material with which to enrich his work. It is to be hoped that he will find, after due study of the text, that he can introduce books of this type into his classes. It is not a book for the untrained teacher who has not had a good course in college mathematics and who has not the ability to do with it what a man like Dr. Swenson can accomplish. Not until our normal schools and teachers colleges can and will insist upon a better knowledge of mathematics, as well as on some proficiency in these subjects which make up the courses in the theory of education, can works of this degree of scholarship be used with success except by teachers of this type of training. It is to be hoped that this book will do something to hasten that time. At present our courses in education demand all the attention of teachers, no doubt imparting a certain amount of knowledge in the making of tests, in administration, and in relating such courses to the lives of children. If, however, these children and adolescents are to have any appreciation of the grandeur and beauty of history, of mathematics, of literature, of science, and of art, the instructor must be better prepared in these fields than is at present too often the case.

If the teacher, at first sight, feels that the sequence followed in this book is not as logical as it should be, that too much abstract drill is sometimes given, or that some of the problems are as antiquated as those against which the author protests in his classes, let him take time to analyze the situation more fully, giving the plan a fair trial in the class room. In any case he will find in the book a source of inspiration to strive for an improvement in his teaching of one of the most interesting, useful, and also beautiful branches of knowledge.—David Eugene Smith.

A Second Course in Algebra. By N. J. Lennes. The Macmillan Company, 1935, IX+377. Price, $1.36.

This book is intended to illuminate and make increasingly meaningful the ideas developed in the Author's "First Course" and also to strengthen and vitalize the connection between algebra and arithmetic.

The purpose of the author is to develop *ideas* and to lead pupils to apply these in varying situations.

There is the usual review of the work of the preceding course that is characteristic of such second courses in algebra. This is questionable, but is the practice of most textbook writers. The idea of review is good if not overdone.

The book is attractive in appearance, is well made and should be of interest to teachers of algebra.

Second-Year Algebra. By Herbert E. Hawkes, Wm. A. Luby and F. C. Touton. Ginn and Company, 1935, VI+360. Price, $1.24.

This book is a revision of the brief edition of the authors' "New Second Course in Algebra." It is designed to follow their "First-Year Algebra" and includes material to cover a standard third semester's work in the subject.

The book begins with a systematic review of the first year's work which may or may not be profitable depending upon how the teacher uses it. New work is presented with the review to keep the work from being mere repetition.

Functions and graphs constitute a desirable chapter and flexibility is indicated to those who wish to take advantage of it.

This new edition of a well known book should be of interest to teachers.

The New Applied Mathematics. By S. J. Lashley and Myrtle F. Mudd. Prentice-Hall, 1934, XXI+450. Price. $1.60.

This very interesting and attractive book is intended to meet an adjustment in the Mathematics curricula of many junior and senior high schools. Even where another text is in use this book would be a good one for the teacher to have on his desk for supplementary material.

The book treats such phases of arithmetic, business, geometry, and algebra as will be worth while for any pupil regardless of what he does later in life.

The authors have worked out a time allotment for those who care to follow it.

Plane Trigonometry. By H. L. Rietz, J. F. Reilly, and Roscoe Woods. The Macmillan Company, 1935, X+142. Price, $2.

This book is designed primarily for freshmen in colleges and technical schools. The authors introduce one important new idea at a time and endeavor to emphasize the essentials. At first they define only the three principal trigonometric functions of an acute angle going to the general angle after a more natural learning process has been stimulated.

A feature is introduced in Chapter V on the graphic representation of the functions by including questions to be answered from the graphs.

Courses of 30 lessons and 45 lessons are outlined by the authors for teachers who prefer not to follow a detailed schedule.

The book has a large number of exercises and problems to meet the difference in ability among students.

The Teaching of Mathematics in the New Education. By N. Kuppuswami Aiyangar. Sold by the author, 1935, VII+420. Price, RS. 5.

The very interesting volume should be read by every teacher of mathematics in the elementary and secondary schools. Even college teachers will find tbe book of value. There are chapters on the following topics:

1. The Nature and Importance of the Question.
2. The Scope and Use of Mathematics. (2 chapters)
3. Mathematical Ability.
4. The Theory of Formal Discipline. (2 chapters)
5. Why Should Everybody Learn Mathematics?
6. The Curriculum in Mathematics.
7. The Organization of the Curriculum.
8. How Should Mathematics be Taught?
9. The Teaching of the Different Branches and Topics in Mathematics.
   a. The Teaching of Arithmetic.
   b. The Teaching of Algebra.
   c. The Teaching of Graphs.
   d. The Teaching of Geometry.
10. Miscellaneous.

Teaching Methods and Testing Materials in Business Mathematics. By R. R. Rosenberg. The Gregg Publishing Company, 1935, XI+259. Price, $0.90.

There are almost no books devoted to the Teaching of Business Mathematics. A few books devote some space to topics treated in Business Mathematics, but such treatment is overshadowed by other topics. Therefore, a book such as this should certainly be welcome to those interested in teaching Business Mathematics.

Each chapter is supplemented with testing material. Teachers should use such material as guides and build up a testing program to suit their own needs.

Such topics as "Life Insurance," "Automobile Insurance," "Accident Insurance," as well as "Installment Buying" have not been included although they would seem to be important.

The book should stimulate others interested in this field to improve their methods.

Business Mathematics—Principles and Practice. By R. R. Rosenberg. The Gregg Publishing Company, 1934, XIII+511. Price, $1.05.

Essentials of Business Mathematics—Principles. By the same author and publisher, 1935, X+310. Price, $0.90.

These two books, of which the second is an abridgement of the first are texts on commercial arithmetic. Since books of this type are not so numerous as algebras and geometries these two books should attract the teachers of commercial arithmetic.

These books are the result of extensive experimental teaching. A close examination of the books leads to the conclusion that the author has given considerable thought in including a maximum of important material. Space will not permit an enumeration of all the topics.

The books contain a considerable amount of problem and test material which should offer the pupils opportunity to do extensive practical work.

Mr. Rosenberg's experience as a Certified Public Accountant qualifies him to speak with authority.

A First Course in Calculus. By Henry L. Slobin and Marvin R. Solt. Farrar and Rinehart, 1935, XI+426. Price $3.

The authors of this new Calculus have tried to present the elements of theory and applications of the subject in as simple a form as possible and at the same time to preserve the rigor in demonstration that is necessary in a *first* course. The book is planned for use both in liberal arts and technical institutions.

While the content of most books on the Calculus is supposed to be more or less standardized, the authors of this text claim enough originality in content to justify the appearance of another text.

The book contains approximately 2,000 exercises of which half are graded and distributed in sets under the separate methods developed to handle the particular exercises.

The text is well illustrated with nearly 200 drawings.

Elements of Projective Geometry. By E. E. Watson and Margaret M. Watson. D. C. Heath and Company, 1935, IV+251. Price, $2.60.

This is a book that should be of interest to all teachers of secondary mathematics who desire to gain some knowledge of a field that has been neglected too much in the training of secondary teachers. The course presupposes on the part of the student a knowledge of elementary (Euclidean) geometry and some trigonometry.

There is an abundance of simple exercises, the principal theorems are used in numerous problems, the principal of duality is introduced early and used constantly and the projective pure geometry is introduced early leaving the metrical phases largely to the latter part of the book.

Descriptive Geometry. By Frank W. Bubb. The Macmillan Company, 1935, IX+230. Price, $2.50.

The book breaks definitely with the traditional method of treating Descriptive Geometry. The Fundamental Space Operations are presented in the first chapter and the student is shown how to analyze space problems. After this the second chapter shows how to draw out the seven operations (already presented) upon paper by plans, elevations, and convenient auxiliary views. The fourth chapter shows how to draw out problems each one of which requires a simple combination of the fundamental operations. The succeeding chapters present numerous applications of the method.

The 1000 exercises in the book will be found ample to serve the practical needs of the classroom.

This is the type of book with which secondary teachers of mathematics should be more familiar.

Business Arithmetic. By Clyde O. Thompson. Prentice-Hall, 1934, X+436. Price, $1.60.

This book is adapted to the varying needs of the large number of pupils who study business arithmetic, and to diverse teaching conditions. It is divided into two parts. Part I deals almost entirely of principles and methods. Part II contains 111 graded assignments and 63 review or test assignments so planned as to admit of flexibility in their use.

The author has tried to give as much practicality and flexibility to the course as is in harmony with good organization and teaching and it looks as if he has succeeded. Moreover, the book is the outgrowth of experimental work which, while it does not guarantee perfection, does add value to the contents.

Analytic Geometry. By R. W. Brink. D. Appleton-Century Company, 1935, VII+331. Price, $2.90.

This book is a complete revision of the author's earlier text. The chapter on Conic Sec-

tions has been simplified and clarified. A short chapter on Curve Fitting has been added with an introduction to the method of least squares, to meet the needs of students in engineering, statistics, or laboratory sciences.

The text is long enough to meet the demands of the more gifted students and yet so organized as to enable the teacher to make adjustments for individual differences in ability.

Throughout the book the emphasis is placed upon logic and method rather than on information. This is as it should be in all mathematics courses. It does not mean that the book is not adequate in its content of information.

Survey of High School Mathematics. By Jos. A. Nyberg. American Book Company, 1935, 394 pages. Price, $1.

The title to this book may be misleading to some because it is not a survey of *all* high school mathematics, but only those parts that are of a more elementary nature. The book is intended to meet the needs of pupils who do not plan to go to college "whose ability, interests, and future are such that the study of even the easiest parts of algebra is of doubtful value." The only question here is whether anyone can be sure at the present time just what such needs are.

This is clearly another terminal course for those pupils who for one reason or another are supposed to be unable to take more than a limited amount of mathematics and yet who need to be "exposed" to the subject. It is doubtful whether any such terminal course will solve the question. However, the material in this text is well selected if only a limited amount is to be given.

If such courses are to be popular other similar books will appear, but the problem of mathematical education for the slow moving group, the able student who cannot continue in school, or the indifferent pupil who needs to be informed will not be solved by one year courses. The organization of mathematics must be sequential and spread out over a longer period of time than one year.

Practical Shop Mathematics. (Elementary) By John H. Wolfe and Everett R. Phelps. McGraw-Hill Book Company, 1935, XI+331. Price, $2.20.

Practical Shop Mathematics. (Advanced) By the same authors and publisher, 1935, XIV+296. Price, $2.20.

These two books are the outgrowth of a course in shop mathematics which was taught under the guidance of Mr. Wolfe at the Ford Apprentice School of the Ford Motor Company. It is intended that these books be used not only in factory schools, trade schools, vocational high schools and the like, but also in high schools to replace the usual geometry course for those students who do not plan to go to college.

The text material is organized so that a student or mechanic can profitably use the books for home study or a reference course.

The record book continues with the application of trigonometry and geometry to shop mathematics. Shop teachers and others will be interested in examining these volumes.

Mathematics for Everyday Use. By John C. Stone and Virgil S. Mallory. Benj. H. Sanborn and Company, 1935, XI+532. Price, $1.28.

This book is intended primarily for that group of students in the high school who do not intend to go to college and who do not succeed well with the traditional type of ninth grade mathematics. It is the outgrowth of an experiment carried out under the supervision of Mr. Mallory in some New Jersey high schools.

Such topics as measurement, graphs, formulas, geometric drawings, numerical trigonometry, business arithmetic, and others intended to furnish the pupil with some kind of training for the problems he will meet in life situations are included.

There is still some question whether a one year book of this terminal nature will solve all of the problems in the ninth grade, but it is clear that more such books are going to appear. The standing of the authors of this volume make it at least worthy of consideration.

A First Course in Algebra. By John C. Stone and Virgil S. Mallory. Benj. H. Sanborn Company, 1936, V+510. Price, $1.36.

Any book prepared by the above authors is sure to be carefully prepared and is sure to have merit. This book is something more than just another traditional algebra because it contains a great deal of the modern spirit of approach. However, the book still contains some material such as the outmoded types of factoring and fractions that we have been trying to eliminate from algebra courses in the last few years. The inclusion of such material even in the newest algebras (and this book is no exception) is due to the influence of the extra-mural examining boards.

The development of the material is psychological, each new topic is based on old concepts and skills and tests and remedial work appear throughout the book.

Teachers of algebra will want to see this book.

Plane Geometry. By Arthur Schultze, Frank L. Senenoak, and Limond C. Stone. The Macmillan Company, 1935, IX+391. Price, $1.40.

This is a revision of one of the best geometries that has appeared during the last generation. While this book retains the distinctive features of the old text like the systematic introduction of the student to original geometric work, it contains some new features that are of interest. Among these are a more thorough and deliberate development of the introduction to formal geometry, geometric constructions and their rôle in the course, a thorough treatment of loci, alternative proofs of selected propositions, correlation of plane and solid geometry where advisable and interesting material in the Appendix.

Teachers of geometry like the reviewer who used the old text with such good results will be interested in this revision.